A Model of Sovereign Default Under Imperfect Information

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Abstract

In this paper, I document that, during the Eurozone Debt Crises, 1.) forecasts of output were persistently biased upwards, 2.) the afflicted countries all saw steep increases in their government debt to GDP ratios and their external government debt to GDP ratios, and 3.) spreads reacted slowly to these increases. I argue that these three facts are related and connect them through a model of sovereign default which features incomplete information with respect to the persistent component of output. I then show that the inclusion of information imperfections allows the model to produce patterns during and before crises which better match the patterns in the data than the benchmark model.

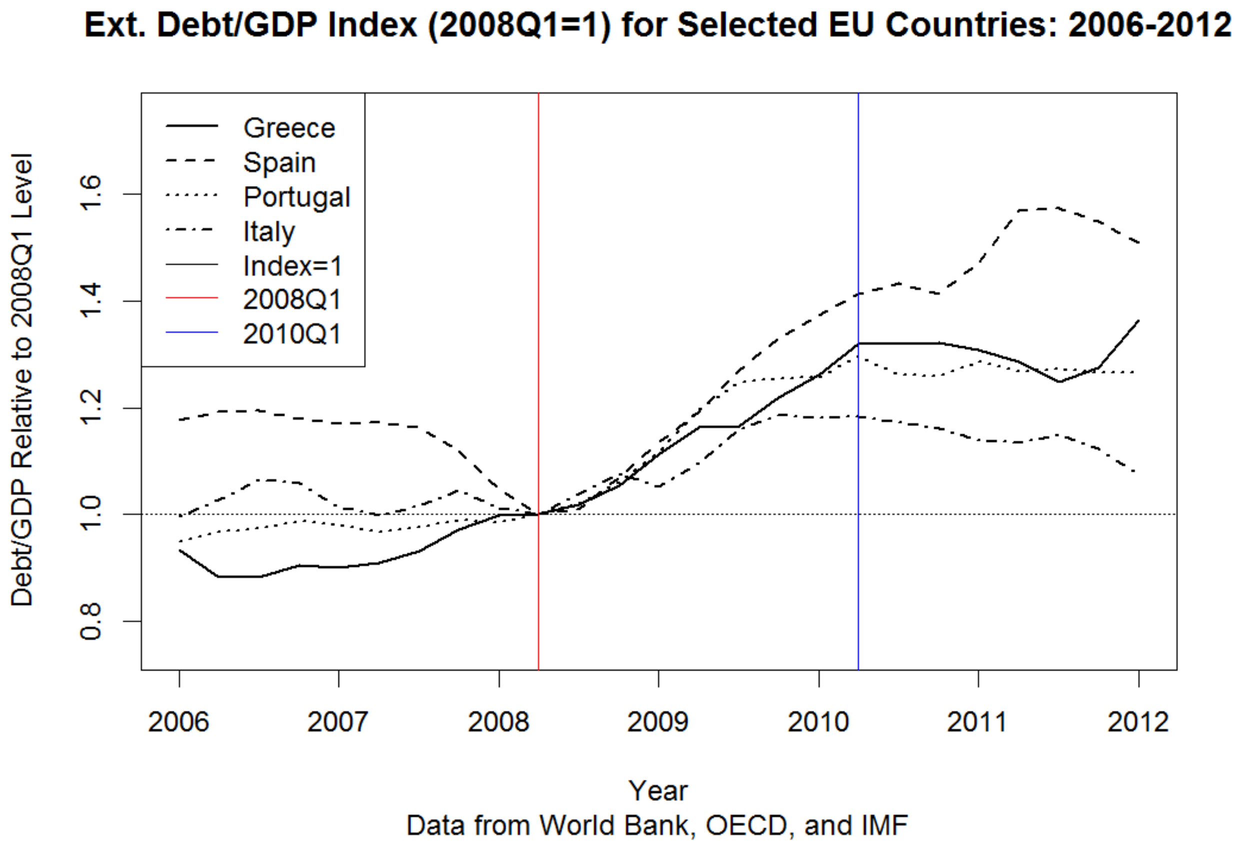
JEL Codes: F34, F41, H63

1 Introduction

Since the publication of the first quantitative implementations, with income and consumption risk, of Eaton and Gersovitz's seminal 1981 paper "Debt With Potential Repudiation" in Aguiar and Gopinath (2006) and Arellano (2008), the literature on sovereign default has grown tremendously. It has made great progress in explaining the patterns in Emerging Market Economies that those two papers sought to explain, namely debt levels, default frequency, spread levels and volatility, and trade balance countercyclicality. A major turning point came with the inclusion of long term debt by Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), which turned out to be a key element to solving the mismatch between the model and the data noted in those papers.

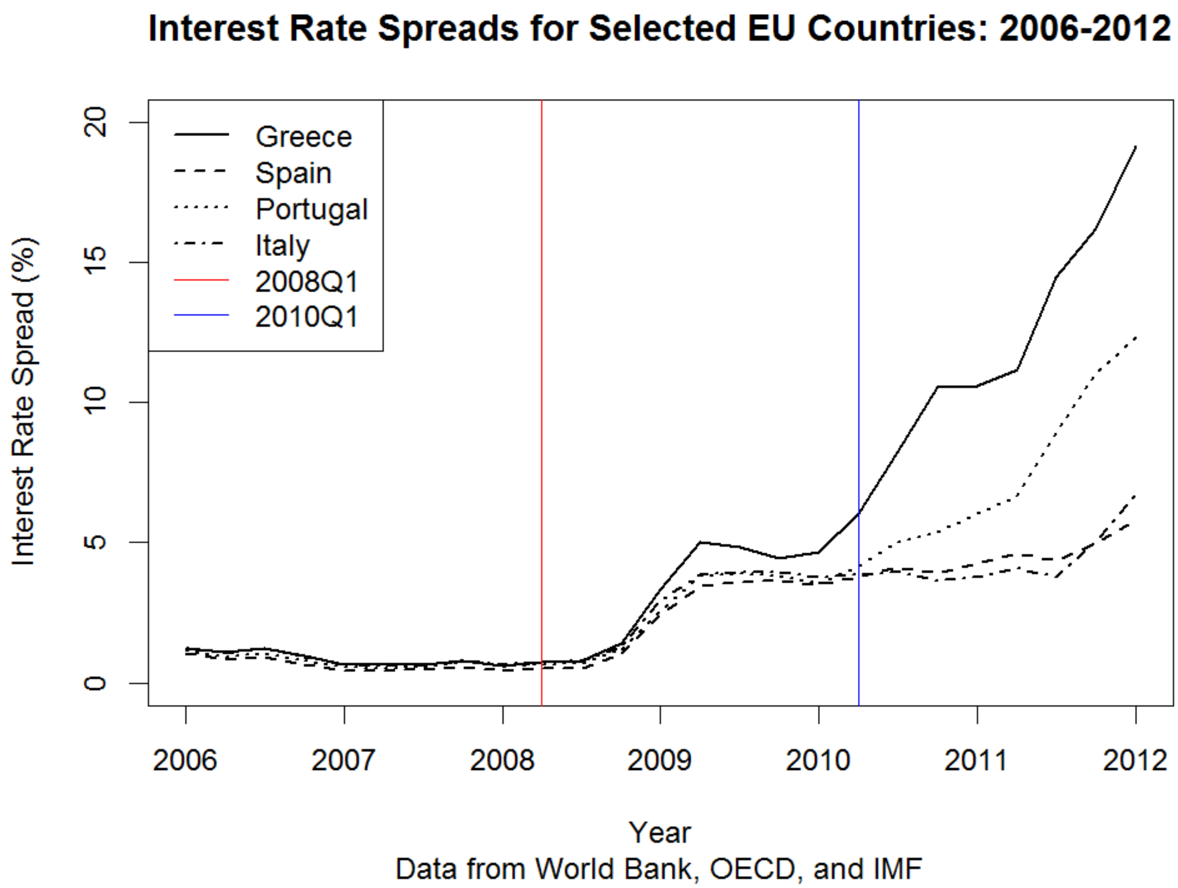
During the development of the literature, the default event of interest was often the Argentinian default. Indeed, all four of the above quantitative papers study this case. Until more recently, somewhat less attention was given to the Eurozone debt crises. These crises differ markedly from the most studied case, Argentina, in that these countries saw rising debt burdens directly prior to receiving their bailouts (whose stated purpose included avoiding default). This phenomenon is not unique to this set of crises – indeed, Benjamin and Wright (2013) document across 112 sovereign default events that mean government debt to GDP rises from a long run mean of 78% to 80% in the year before default and 90% in the year of default. That said, it is particularly pronounced in the European cases, especially in those of Greece. Figure 1 shows external government debt to GDP for Greece, Italy, Spain, and Portugal from 2006 to 2012, normalized by their levels at the beginning of 2008.

Figure 1:



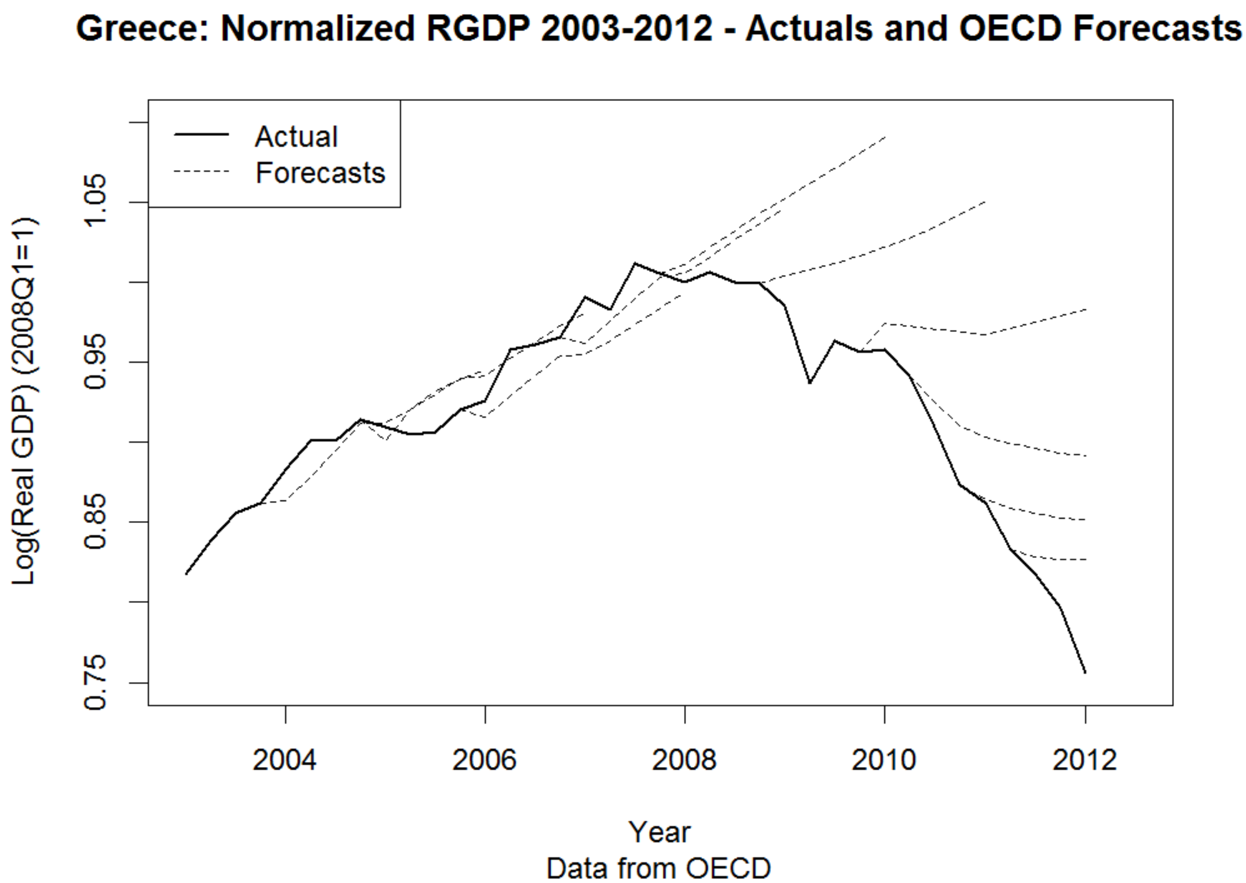
All four countries show a sharp rise in external debt to GDP over the next two years, as well as over the first year with the exception of Italy. In spite of this, interest spreads for none of the countries show a particularly pronounced rise until at least the first quarter of 2009. Figure 2 shows the interest rate spread for government debt relative to German debt with a maturity of less than a year for the same countries and period.

Figure 2:



Finally, forecasts of output for the same countries during this period show a persistently one-sided bias, a classic indicator of imperfect information. Paluszynski (2019) documents this for a broad set of forecasts for these four countries. Figure 3 plots the actual sequence of quarterly real GDP for Greece as well as quarterly OECD forecasts from 2003 to 2012.

Figure 3:



In this paper, I will argue that these three facts are connected. In short, decisions related to the future depend, of course, on beliefs about the distribution of future events. The optimism of forecasts, especially during 2008 is therefore reflected in the interest rates that investors offered the Greek government. Furthermore, the government sees a sharp drop in output and, impatient or not, would like to smooth consumption. Like investors, it also expects the state of the economy to quickly improve, and therefore sharp increases in borrowing today should not lead to particularly low consumption in the future. For these reasons, it takes the investors up on their offer of relatively low interest rates for relatively large amounts of new borrowing. I will show that a model which incorporates information imperfections captures this interaction and that this allows it to match the dynamics of key variables in the time leading up to a default. Moreover, it provides a marked improvement over a perfect information benchmark.

This benchmark will be a standard model of sovereign default. By standard model, I mean one with the following five features:

1. Small open economy with a representative consumer that has expected utility preferences over infinite streams of consumption.

2. Exogenous output. Deviation from trend follows an $AR(1)$ process and/or an i.i.d. one. Trend growth may be constant or follow an $AR(1)$ process.

3. The government has access to a single asset.

4. Creditors are risk neutral and competitive.

5. There is no recovery for creditors after a default.

Such standard models of sovereign default struggle to produce rises in debt to GDP prior to crises. For example, Chatterjee and Eyigungor (2012) and Bocola et al. (2019), and Arellano and Bai (2017) belong to this group (although the calibration targets for the last two are debt service, rather than level, since they use a one period bond). It is well known that allowing nonzero recovery in a model with a long term asset can allow for steep rises in debt to GDP prior to a default, especially in the preceding period. In order to distinguish the effect of information imperfections from that of nonzero recovery, I will shut down the recovery channel entirely.

In this paper, I follow Paluszynski (2019) and Chatterjee and Eyigungor (2019) in constructing a version of the Eaton Gersovitz model with incomplete information. Both papers, however, restricted the unobserved state to taking one of two values. While it is not Paluszynski's primary focus (his main concern is explaining spread volatility in excess of the mean spread), he does note how debt to GDP rose prior to the European crises and that a two state model, even with imperfect information, struggles to fully reproduce it. This paper also builds on the expanding literature relating to the European sovereign debt crises. The model of maturity choice in Bocola and Dovis does indeed produce rising debt to GDP ratios along the sequence of shocks implied by the data for Italy without assuming nonzero recovery. However, the remainder of the model differs substantially from the "standard" model I describe above. The model in this paper provides a way to substantially close the gap with near minimal deviation from the "standard" model.

2 Model

Time is discrete and infinite. There is a small open economy with a representative consumer and a benevolent government who have identical expected utility preferences over streams of consumption given by:

$$E[\sum_{t=0}^{\infty} \beta^t u(c(s^t))] \tag{1}$$

where $\beta \in [0, 1)$ and $u : \mathbb{R}_{++} \to \mathbb{R}$ is a continuously differentiable function which is strictly increasing, homogeneous of degree $1-\gamma$, and satisfies $lim_{c->0}u(c) = -\infty$ and $lim_{c->0}u'(c) = +\infty$. Income is the combination of a permanent process g_t, a transitory process x_t, and an i.i.d. process m_t. Specifically, it is given by:

$$Y_t = G_t * (x_t + m_t) \qquad\qquad G_t = g_t * G_{t-1} \tag{2}$$

All three are assumed to have bounded support. The i.i.d. process m_t is assumed to be a continuous random variable with a well defined density. It is included primarily for computational reasons (to ensure convergence) and will not be discussed in detail.[1] For simplicity, I further assume that x_t and g_t are separately observed.[2] x_t and g_t in turn are each a combination Markov Process and a mean zero iid process:

$$x_t = f_x(z_{x,t}, \epsilon_{x,t}) \qquad\qquad g_t = f_g(z_{g,t}, \epsilon_{g,t}) \tag{3}$$

The laws governing transition probabilities of the z processes and the ϵ processes are known to all agents. x_t and g_t are also observed by all agents, but their decomposition into their components is not. Agents then form beliefs about the current value of the z processes and update them following Bayes Law. Let Γ_T denote the posterior beliefs about the current

[1] For details of its role, see Chatterjee and Eyigungor (2012).

[2] For a discussion of the resulting filtering problem when this is not the case, see Boz et al. (2011).

values of the persistent components after observing all signals up to time T (inclusive). Since beliefs are updated using Bayes Law, Γ_T is a sufficient statistic for any history of observations, and therefore beliefs are a First Order Markov Process. Let $T(\Gamma, x, g)$ denote the operator which updates beliefs Γ upon witnessing x and g.

The government may borrow on international markets using a long term bond, can default, and cannot commit not to default. When it defaults, it enters financial autarky and suffers an output penalty $G_t\phi(g_t, x_t)$. It exits autarky at constant rate θ. International lenders are risk neutral and discount at rate $\frac{1}{R}$ with $\beta R < 1$. Following Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), I use a probabilistic characterization of maturity. Specifically, each bond matures with constant probability λ each period. With complementary probability, the bond instead pays a coupon κ.

I now move to the recursive, detrended characterization of the government's problem (Appendix 1 shows that this is equivalent to the original problem). Let $s = (x, g, m)$ denote the state of income realizations related variables. Under repayment the government's problem is:

$$W^R(\Gamma_-, s, a) = max_{c,a'}\ u(c) + \beta E[g'^{1-\gamma} W(\Gamma, s', \frac{a'}{g'})|\Gamma] \tag{4}$$

such that

$$c + q(\Gamma, a')(a' - (1-\lambda)a) \leq (x+m) + (\lambda + (1-\lambda)\kappa)a \tag{5}$$

$$\Gamma = T(\Gamma_-, g, x) \tag{6}$$

Under default, the government's value is given by:

$$\begin{aligned} W^D(\Gamma_-, s) &= u(x + m - \phi(g, x)) \\ &\quad + \beta E[\theta g'^{1-\gamma} W(\Gamma, s', 0) + (1-\theta) g'^{1-\gamma} W^D(\Gamma, s')|\Gamma] \qquad (7) \\ \Gamma &= T(\Gamma_-, g, x) \qquad (8) \end{aligned}$$

The government's problem at the beginning of the period when not in default is then:

$$W(\Gamma_{-}, s, a) = max_{d\in\{0,1\}}\ (1-d)W^{R}(\Gamma_{-}, s, a) + W_{0}^{D}(\Gamma_{-}, \underline{s}) \tag{9}$$

where $\underline{s} = (x, g, \underline{m})$.

2.1 Equilibrium

An equilibrium for the above environment consists of:

1. Value functions W^R, W^D, and W for the government;
2. Policy functions $c^\star, a'^\star, d^\star$ for the government;
3. Prices q

which satisfy the following conditions:

1. Given W, $c^\star$ and $a'^\star$ solve the problem in (4) and W^R is the resulting value function.
2. Given W, W^D satisfies the recursion in (7).
3. Given W^R and W^D, $d^\star$ solves the problem in (9) and W is the resulting value function.
4. Prices q satisfy:

$$\begin{aligned} q(\Gamma, a') = \frac{1}{R} E\Bigg[&(1 - d^\star(\Gamma, s', a')) \\ &\Big(\lambda + (1-\lambda)\Big(\kappa + q(T(\Gamma, x', g'), a'^\star(\Gamma, s', a'))\Big)\Big)|\Gamma\Bigg] \end{aligned} \tag{10}$$

2.2 Theoretical Results

Apart from the separation of current income values from the distribution of future income values, this setting is essentially identical to the one considered in Chatterjee and Eyigungor (2012). For this reason, the following results are essentially immediate consequences of their counterparts in that paper.

1. Existence of an upward sloping equilibrium price function;
2. For any price function, existence, uniqueness, and monotonicity (in a, whenever it is an argument) of all value functions;
3. For any price function, monotonicity of $d^\star$ in a (decreasing) and m (decreasing);
4. For any price function, monotonicity of $a'^\star$ in m (increasing);
5. For any increasing price function, monotonicity of $a'^\star$ in a (increasing);
6. For any price function, given Γ, monotonicity of $d^\star$ in x (decreasing) and $\frac{a}{g}$ (decreasing).
7. For any price function, given Γ, monotonicity of $a'^\star$ in x (increasing);
8. For any increasing price function, given Γ, monotonicity of $a'^\star$ in $\frac{a}{g}$ (increasing).

The first 5 are direct corollaries of their counterparts in Chatterjee and Eyigungor (2012). The seventh is reached by extending the proof of 4 to include other variation in income that does not affect prices or continuation values. The eighth is reached by extending the proof of 5 to variation in asset level unassociated with detrended income level which does not affect prices or continuation values. The sixth is reached by extending the proof of the 3 in both the above manners. The first two do require the following condition on growth rates:

$$\forall \Gamma,\ \beta E[g'^{1-\gamma}|\Gamma] < 1 \tag{11}$$

3 Quantitative Analysis

3.1 Calibration

The model was calibrated using data on the economy and government borrowing activities of Greece. While Greece's debts were not restructured until March of 2012, the first Greek bailout occurred in May of 2010 with the stated purpose of avoiding an imminent default. For this reason, I exclude from use in estimation or calibration most data from after the first quarter of 2010. The income process was assumed to have the following form[3]:

$$ln(x_t) = ln(z_t + \epsilon_t) \qquad ln(g_t) = ln(\bar{g}) \tag{12}$$

$$z_t = \rho z_{t-1} + \eta_t \qquad \eta_t \sim N(0, \sigma_\eta^2) \qquad \epsilon_t \sim (0, \sigma_\epsilon^2) \tag{13}$$

The parameters $\rho, \bar{g}, \sigma_\eta^2, \sigma_\epsilon^2$ were estimated using OECD data on Greek Real GDP from 1975Q1 to 2010Q1 using standard state space methods. The resulting estimates are:

Table 1: Estimated Income Process Parameters

Parameter	**Value**	**SE**
ρ	0.9715	0.0181
σ_η	0.0145	0.0023
σ_ϵ	0.0171	0.0019
$ln(\bar{g})$	0.0038	0.0006

In both the model with imperfect information as well as the perfect information benchmark, it is computationally necessary to have an i.i.d. continuous shock which is added to income (the m shock mentioned above). In both cases, I parametrize this shock as $m \sim TN(0, \sigma_m^2, -\bar{m}, \bar{m})$ with the truncation points given by $\bar{m} = 2\sigma_m$. In the imperfect information case, I set $\sigma_m = 0.003$, a value just large enough that the computational algorithm converges for a wide variety of parameters. To compensate for the extra variation in income this induces, I set $\hat{\sigma}_\epsilon = sqrt(\sigma_\epsilon^2 - \sigma_m^2)$. In the perfect information benchmark, I set $\sigma_m = \sigma_\epsilon$ and $\hat{\sigma}_\epsilon = 0$ – i.e. the persistent component of output is observed perfectly, but the total income process itself is unchanged. This yields a pair of models where every fundamental of the economies are identical, except for the information structure.

This implementation of imperfect information appears to produce that are about as accurate as OECD forecasts from 2004 to 2010. Therefore, it seems that the government in the imperfect information model as approximately as much information as forecasters at the time did. The table below contains root mean square error values for forecasts $1 - 9$ steps ahead.

[3]Earlier versions of this paper included a version of the model with persistent shocks to growth instead of to the deviation from trend. They are excluded from this version because that version of the model cannot match the relevant moments in the data. Those results, however, are available on request.

Table 2: Accuracy of RGDP Forecasts For Greece: 2004Q2-2010Q1

Steps	**Model: RMSE**	**OECD: RMSE**
1	0.019	0.019
2	0.025	0.028
3	0.031	0.028
4	0.037	0.041
5	0.042	0.044
6	0.047	0.060
7	0.050	0.060
8	0.053	0.065
9	0.056	0.071

The maturity and coupon parameters of the asset structure were estimated using data on the individual bond issues that were included in the March 2012 restructuring. To determine the portfolio of bonds on which Greece defaulted (for the purposes of this calibration), I use calculate bond by bond haircuts using a method broadly similar to that recommended by Sturzenegger and Zettelmeyer (2008). Appendix 3 contains a detailed description of the procedure. I then calculate the Macaulay Duration and weighted average coupon rate (κ) for the resulting portfolio of avoided obligations. Since the Macaulay Duration of the bond in the model is $MD = \frac{R}{R-(1-\lambda)}$, I set $\lambda = \frac{R}{MD} - (R-1)$.

The utility function was set to be CRRA:

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & \gamma \neq 1 \\ ln(c) & \gamma = 1 \end{cases} \tag{14}$$

The risk aversion parameter γ was set to 2, the standard value in the sovereign default literature. The default cost function was set, following Chatterjee and Eyigungor (2012), to be:

$$\phi(x) = max\{d_0 x + d_1 x^2, 0\} \tag{15}$$

The reentry parameter was set to 0.0385, also following Chatterjee and Eyigungor (2012). The international interest rate r was set to the standard value of 0.01. The full set of non-income process parameters set outside the model is:

Table 3: Non-Income Parameters

Parameter	Value	Notes
γ	2.00	Standard
r	0.01	Standard
θ	0.0385	CE 2012
λ	0.0318	Macaulay Duration of Defaulted Debt
κ	0.0115	Average Coupon of Defaulted Debt

The discount factor β and the two default cost parameters were selected by jointly matching the mean external debt to GDP ratio from 2003Q2 to 2010Q1, the mean spread from 2001Q1 to 2010Q1, and the volatility of spreads from 2001Q1 to 2010Q1. Spreads were calculated using the difference between the interest rate on Greek government bonds reported in the IFS and the interest rate on German T-Bills (or very short duration debt) reported by the Bundesbank. Since the model does not include renegotiation (and assumes a haircut of 100%), I follow Chatterjee and Eyigungor (2012) in using the convention that the debt in the model only corresponds to the "unsecured" portion of debt in the data. To calculate this, I multiply the raw external debt to GDP ratio by the average participation rate and haircut. The three calibrated parameters are:

Table 4: Calibrated Parameters

Parameter	Imperfect Information Value	Perfect Information Value
β	0.983	0.983
d_0	-0.286	-0.264
d_1	0.361	0.335

The discount factors required to match the targeted moments in both models are significantly higher than is typical in the sovereign default literature. However, they are similar to those found in other more recent work on the European crises, such as Paluszynski (2019) who uses $\beta = 0.987$ for a model fit to Portuguese data and Bocola et al. (2019) who use $\beta = 0.98$ for a model fit to Spanish data.

The model was estimated using Simulated Method of Moments. It was solved in Julia using value function iteration. The belief and income spaces were discretized into 100 points each spanning three standard deviations around their mean[4]. The asset space was discretized into 501 points between -2.5 and 0.0. The level of -2.5 was chosen to be low enough that it was never binding in equilibrium. The routine iterated on the combination of (4), (7), (9), and (10) until the sup norm distance between successive iterations was less than $1e-6$ for all objects.

4 Results

Table 5 contains the values of the targeted moments.

Table 5: Calibrated Moments

Moment	**Data**	**Imperfect Information**	**Perfect Information**
Mean Debt/GDP	33.2%	33.2%	33.3%
Mean Spread	2.10%	2.09%	2.10%
Spread Volatility	1.33%	1.33%	1.33%

Table 6 contains values of moments not directly targeted. Most of the moments which the model misses significantly are those involving the trade balance, and those moments in the data are rather sensitive to the sample period (the values below are for 1975Q1-2010Q1, whenever possible).

Table 6: Nontargeted Moments

Moment	**Data**	**Imperfect Information**	**Perfect Information**
$\rho(r, y)$	-0.475	-0.537	-0.498
$\rho(r, TB)$	0.277	0.629	0.553
$\sigma(c)/\sigma(y)$	1.085	1.091	1.078
$\rho(TB, y)$	-0.149	-0.204	-0.071
$\sigma(TB)$	0.037	0.016	0.020
Default Rate	2.85%	1.81%	1.82%

In most dimensions, the two models are broadly similar. Indeed, for most of the nontargeted moments, it is generally the case that the two models either are both close to the data or both far away. Even the parameters required to match the targeted moments are extremely

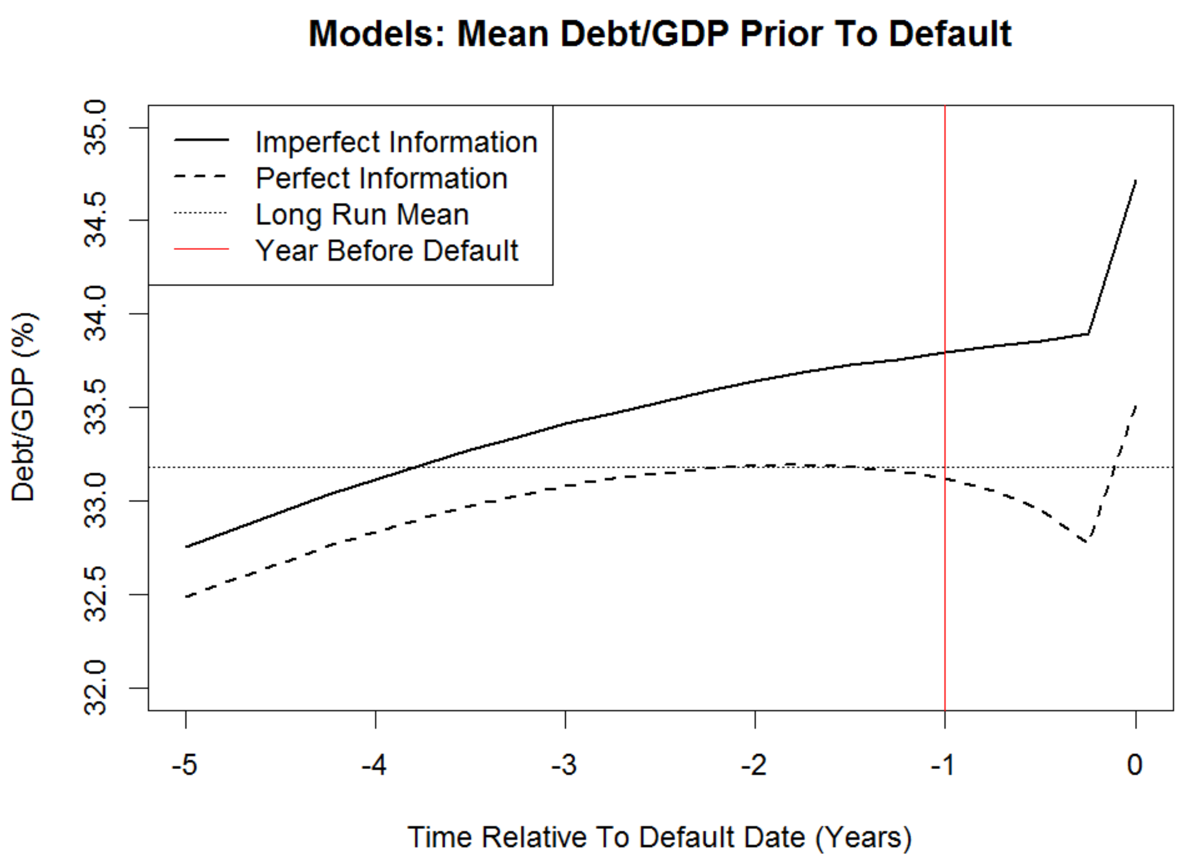
[4]For more details on the discretization method, see Appendix 2

similar. In many ways, in the long run, the models produce extremely similar behavior. Since all agents are Bayesians with respect to updating their beliefs, their beliefs are, on average, correct in the imperfect information case (they are, of course, also correct on average in the perfect information case). This yields a strong long run correspondence between the full set of equilibrium objects in the models.

4.1 Defaults

They differ markedly, however, on the patterns of borrowing which precede a default. Below, I plot the paths of debt to GDP in both models during the 5 years preceding a default.

Figure 4:



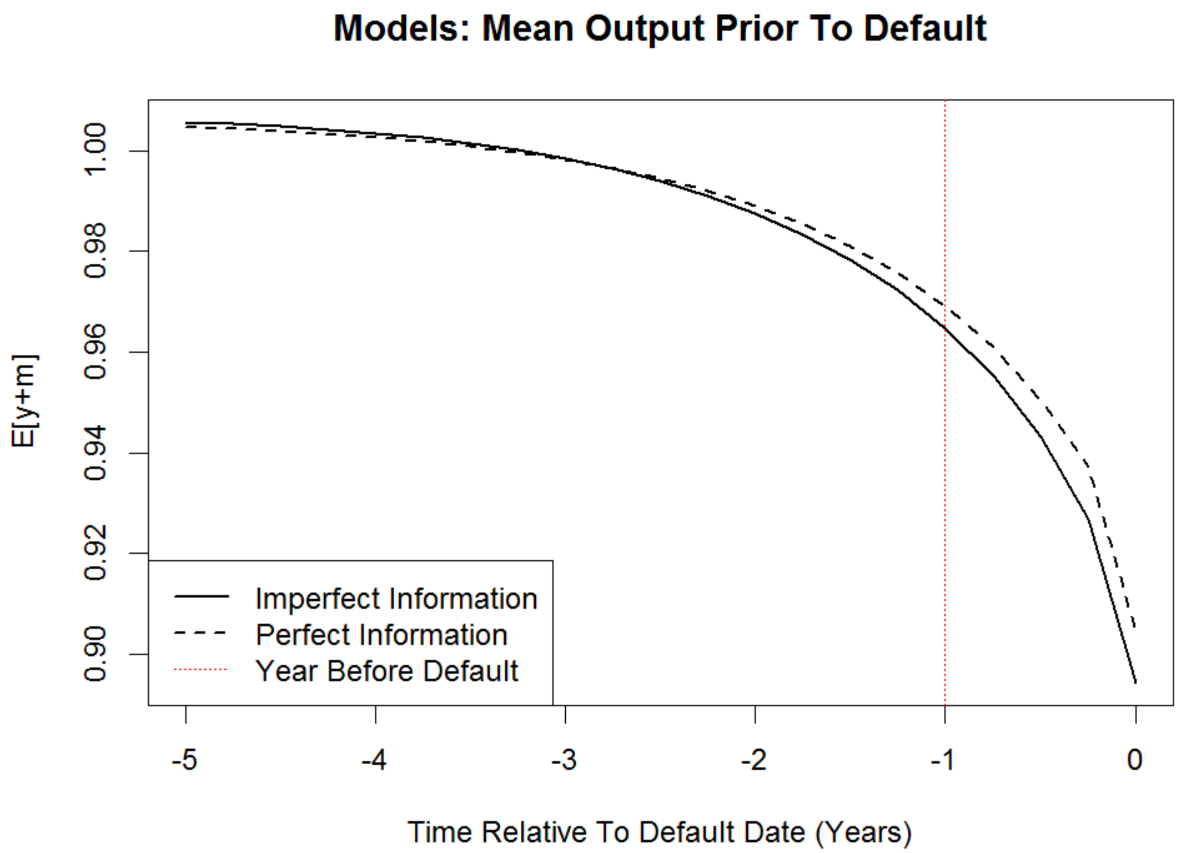
I have included the period of default in order to compare the results with the numbers of

Benjamin and Wright (2013) mentioned earlier[5]. In the model with imperfect information, debt to output climbs steadily throughout the period, passing the mean almost 4 years before the default, before jumping up sharply in the period of default. It ends up 1.5% higher than the mean and 0.92% higher than its level one year before. On the other hand, in the perfect information model, average debt to output barely passes the mean almost two years before default before steadily falling until the period of default itself, when it jumps up. It ends up 0.3% higher than the mean and 0.4% higher than its level one year before.

This difference does not emerge because the sequence of income shocks which precedes a default differs significantly between the two models. Figure 5 plots the path of average output leading to a default. While the perfect information case does display a smaller drop overall and higher endpoint, the largest difference between the two paths is less than a single standard deviation of either the innovation to the persistent component of output or the sum of the i.i.d. processes.

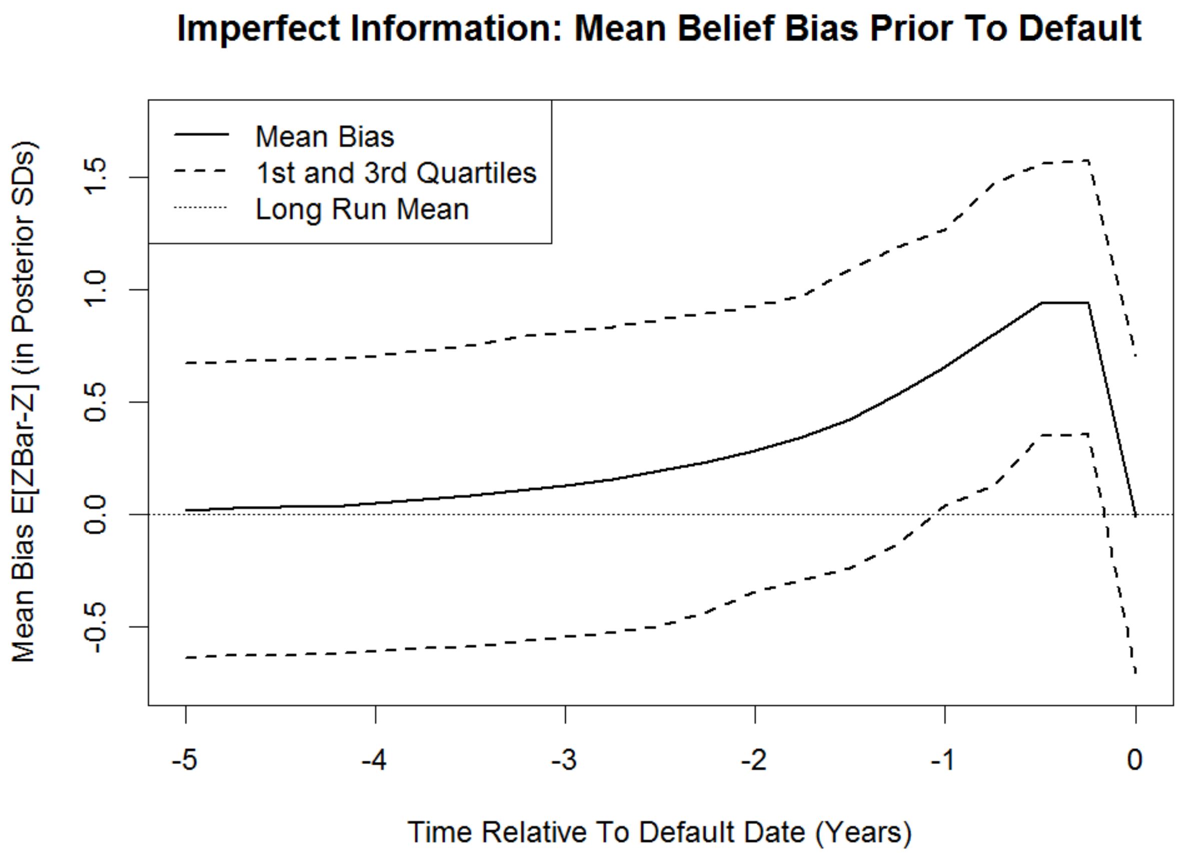
[5]For both models, debt to output in the period of default is calculated as beginning of period debt divided by unpenalized output, which is the least generous measure possible

Figure 5:



Instead, the difference emerges because defaults in the model with imperfect information require more, on average, than just a very bad sequence of income realizations. They require persistently optimistic forecasts of the future, based on biased beliefs. In the period of default, that optimism vanishes as a particularly poor income shock forces beliefs to adjust downwards to an essentially unbiased level. Figure 6 plots the average bias of beliefs prior to a default in the imperfect information model.

Figure 6:



That bias leads to lenders offering the government relatively good prices. For its part, the government sees a poor income shock today but expects times to improve tomorrow. The mix of the government's relative impatience, high marginal utility today, and relatively good prices lead to the choices which cause debt to output to rise. However, only one of those three ingredients is specific to the model with imperfect information. In the model with perfect information, the government is also relatively impatient (in fact, by almost the exact same amount) and has relatively high current marginal utilities. However, beliefs in the model with perfect information are never biased, so prices fall much faster as the state of the economy worsens. Figure 7 plots the average sequence of price functions faced by the government in both models two years before default, one year before default, one quarter before default, and in the quarter of default:

Figure 7:

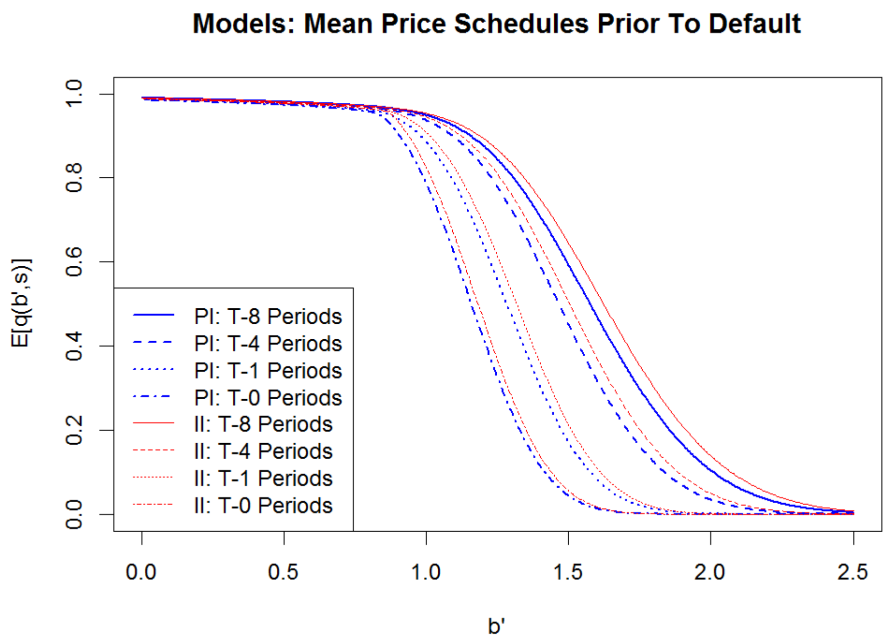
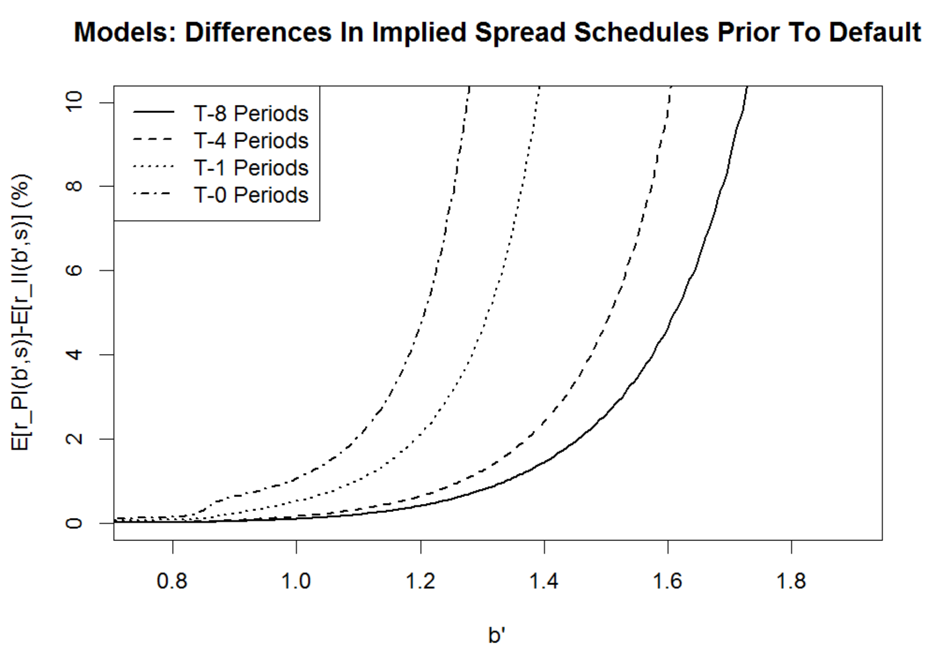


Figure 8 plots the difference in implied spreads at the same set of horizons:

Figure 8:



In terms of the mean of realized spreads along paths leading to default, the models are essentially identical, again (the largest difference, 0.13% occurs two periods before default). It is not that the government in the imperfect information model is choosing to take bigger risks than its perfect information counterpart. Indeed, it is actually making choices involving approximately the same total amount of risk, on average. The difference is that those same risks are associated with more borrowing.

In order to compare the magnitudes of the debt to output patterns explored above to those of Benjamin and Wright, it is necessary to normalize them by the reference point. Their 78% long run mean, 80% one year prior to default, and 90% in the year of default translate to a debt to output ratio in the period of default which is 15% above average and 12.5% higher than one year previous. Converting the numbers produced by the models in the same way yields 4.6% above average and 2.7% higher than one year previous for the imperfect information case and 1% above average and 1.2% higher than one year previous for the perfect information case. The model with imperfect information explains more than four times as much of the patterns in the data by the first measure and more than twice as much by the second measure. That said, the total share of the target rise produced by the imperfect information model is about 30% by the first measure and 22% by the second.

While these numbers may seem somewhat small, and their differences smaller, it is important to keep in mind 1.) that these are nontargeted moments for both models as well as 2.) how very similar the underlying models are. Preferences of all agents are essentially identical (exactly with respect to flows and risk – almost with respect to time), the income processes are identical, and the penalties for defaulting are quite similar. Neither model has recovery and therefore high dilution incentives during periods of high risk. The only thing that differs is the structure of information, and this difference is enough to deliver fiscal policies which become countercyclical in extreme cases, in spite of the relative impatience of the government.

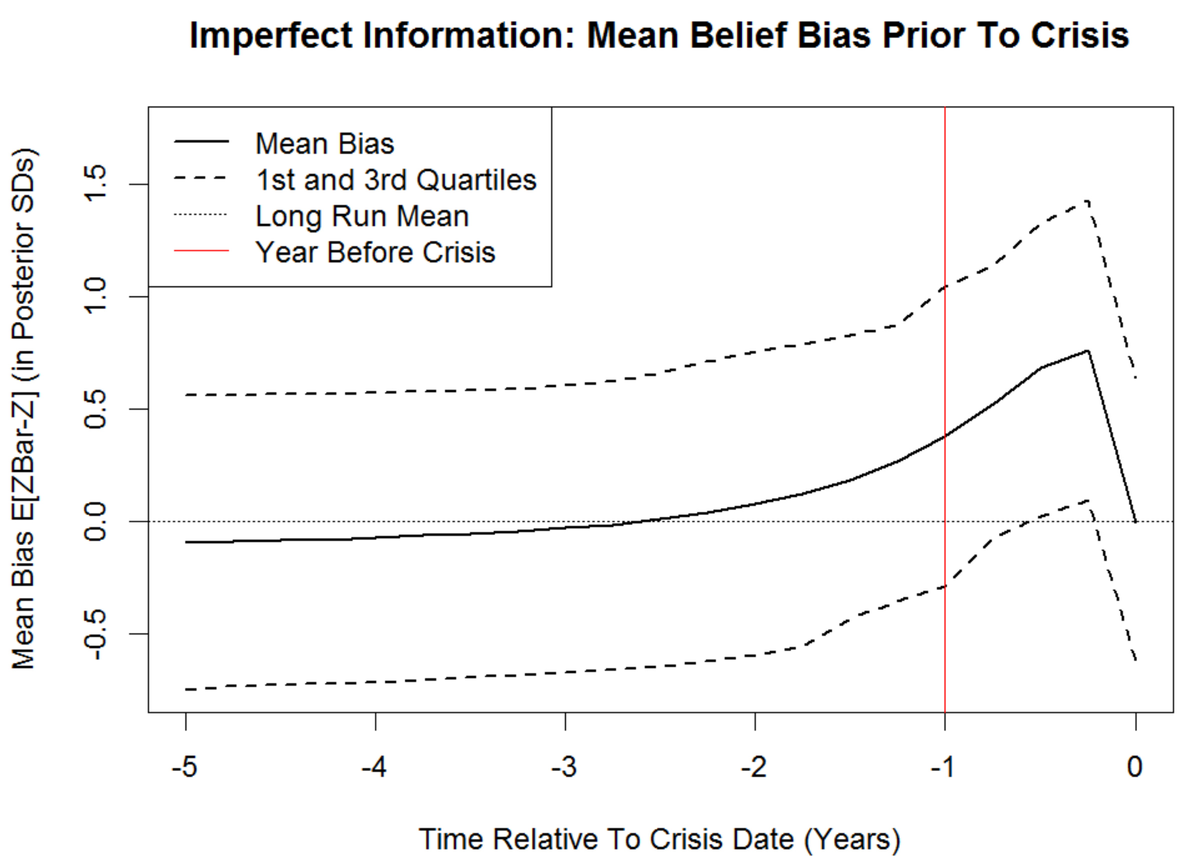
4.2 Crises

Instead of restricting the event of interest to actual defaults, we might instead consider a broader set. Bocola, Bornstein, and Dovis define a the onset of a debt ”crisis” to be a period in which:

1. either a default occurs or the interest rate spread rises above its mean plus one standard deviation;
2. the economy has not recently experienced either of the conditions in 1..

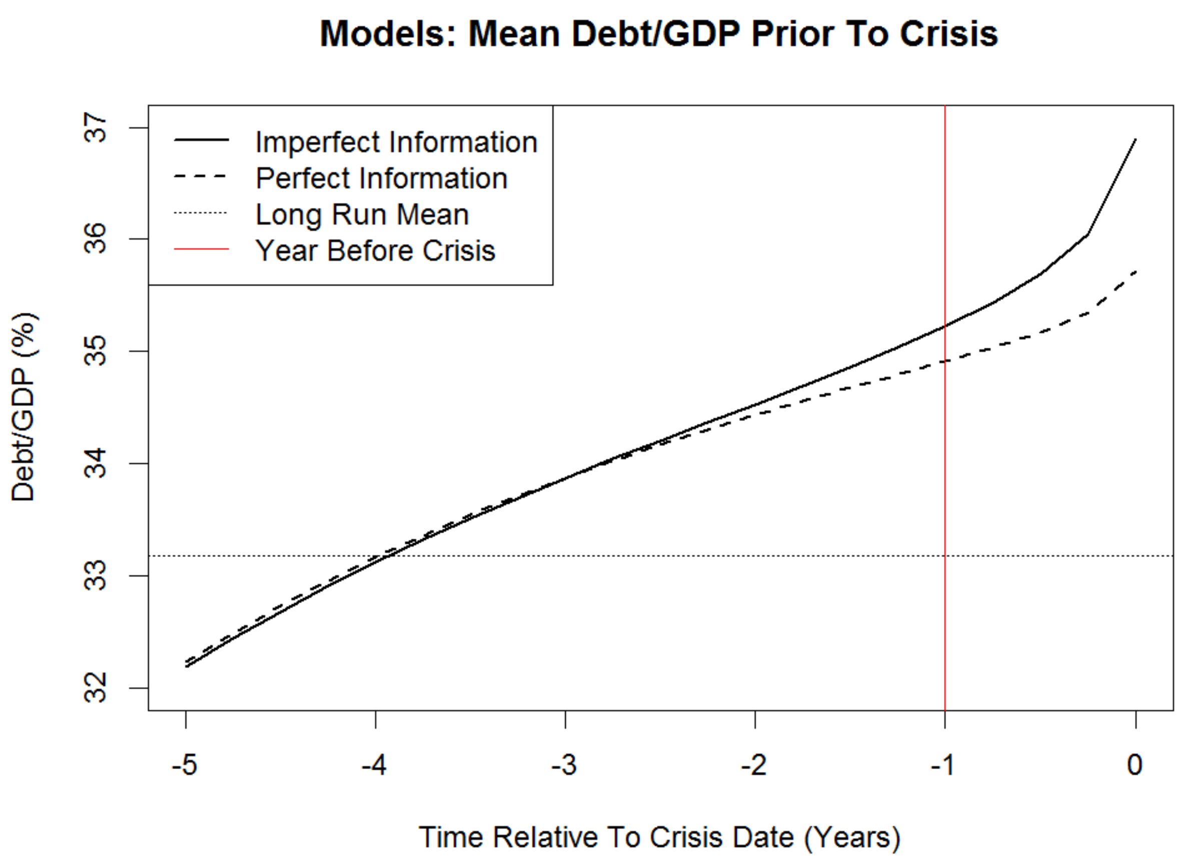
The pattern of belief bias which precedes such a crisis is essentially a scaled down version of the pattern which precedes a default (it peaks at 0.75 posterior standard deviations instead of at 1). Figure 9 plots it over the 5 years preceding a crisis.

Figure 9:



The pattern of debt to GDP in the lead up to a crisis looks quite similar as well, although the magnitudes now make the differences between the model predictions much more apparent:

Figure 10:



Before this broader set of events, both models predict relatively steady average rises in debt to output. Until slightly more than two years before the onset of the crisis, the mean paths of debt to output are almost identical. Then, at essentially the exact same time that the belief bias begins rising away from zero, the path under imperfect information begins separating and rising more quickly. This divergence accelerates throughout the rest of the lead up to the crisis. In the first period of the crisis, the model with imperfect information predicts debt to output 3.7% higher than average, with an increase of 1.7% over the previous year while the model with perfect information predicts debt to output 2.5% higher than average, with an increase of 0.8% over the previous year.

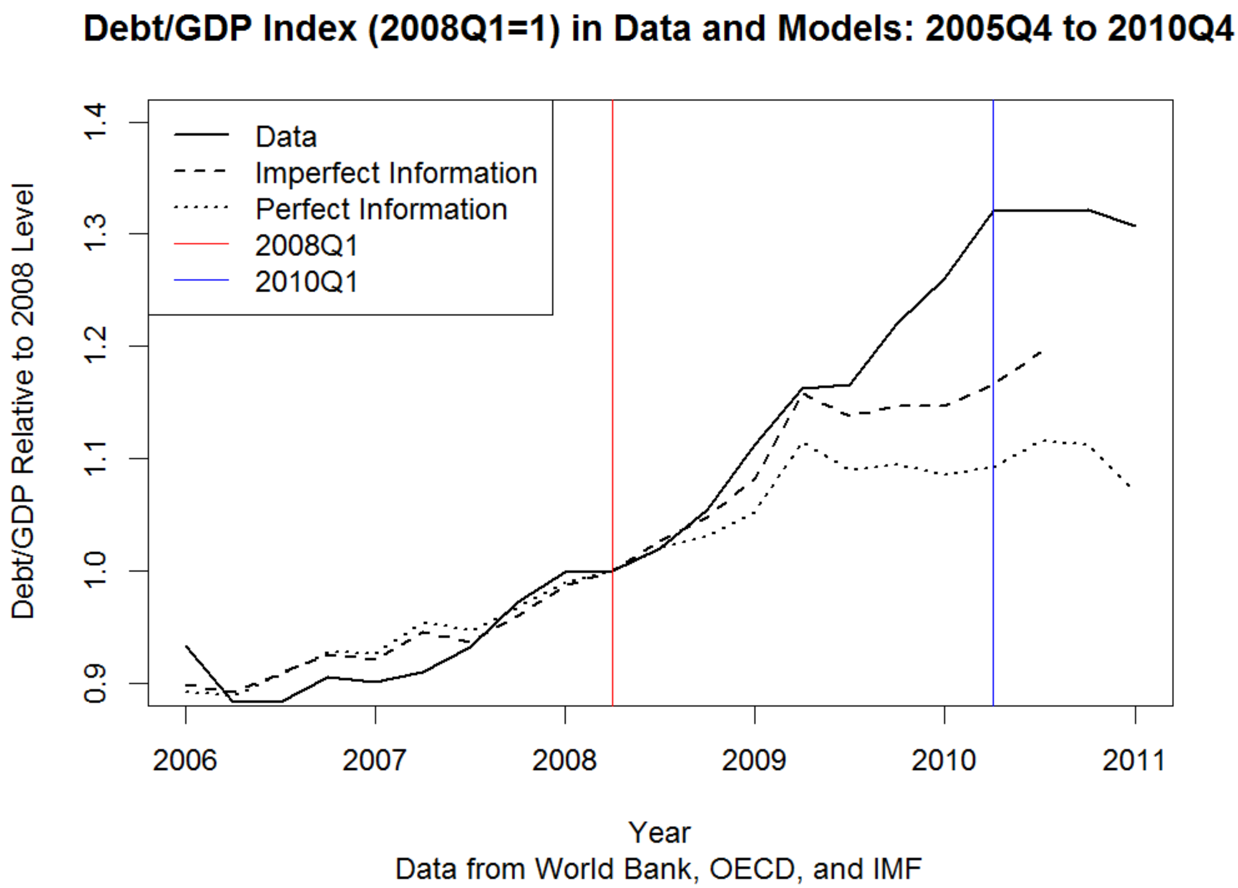
Translated again to proportional terms, those become a debt to output ratio 11.2% above average and 4.7% higher than a year previous in the imperfect information case and a debt to output ratio 7.7% above average and 2.3% higher than a year previous in the perfect information case. If we consider these the correct analogue to the numbers produced by Benjamin and Wright (2013) (which has the advantage of not requiring a model definition of debt to output in the period of default), the model with imperfect information still significantly outperforms its perfect information counterpart. Furthermore, the total share of the target rise produced by the imperfect information model is about 73% by the first measure and 38% by the second.

4.3 Realized Shocks In Data

While allowing for information imperfections does help substantially close the gap between the model with perfect information and the data in terms of average outcomes, the magnitudes involved are relatively small compared to the increase in Greek debt to GDP from 35.1% in 2008Q1 to 40.7% in 2009Q1 (the first quarter in which spreads rose above their mean plus one standard deviation) to 46.3% in 2010Q1, on the eve of the country's first bailout. Similarly, the sequence of income shocks which struck Greece in during this period were unusual both in terms of how quickly the drop occurred as well as how high above trend output was when it started.

In order to give both models a fair chance to match the data under this sequence of extreme shocks, I initialize both of them in the first period for which I have data on external debt and simulate their behavior throughout the entire sample period. Since neither model reaches the extremely high levels of debt to output witnessed in the last year of the sample period, neither can match the average debt to output during the period for these shocks. For this reason, and since the purpose of this paper is to explore the pattern and timing of changes, rather than absolute levels, I normalize all three series by their values in 2008Q1. Figure 11 plots the result.

Figure 11:



The model with imperfect information does quite well in matching the rate at which the Greek government accumulated debt over the first year of the crisis. Thereafter, it does somewhat worse. The model predicts certain default in the third quarter of 2010, which seems plausible given that the stated purpose of Greece's first bailout in May, 2010 was to avoid an imminent default. In terms of the response of debt to output, the model with perfect information performs worse over the first year, and at best the same as the version with imperfect information from the end of the first year to the end of the second year. It predicts defaults occurring as early as the second quarter of 2010 (1.4%), with most occurring in the third quarter of 2010 (85.0%), but some occurring in the fourth quarter of 2010 (13.3%) and a small fraction as late as the first quarter of 2011 (0.4%).

5 Conclusion

The purpose of this paper is not to show that adding only information imperfections to a simple, benchmark sovereign default model can fully close the gap between the predictions of that benchmark model regarding patterns of debt to GDP prior to crises and/or defaults and observed patterns in the data. Its purpose is to explore whether there exists a relationship between the optimistic forecasts of output growth on the eve of the European Debt Crises and the rapid increases in debt to output that occurred in the afflicted countries. To that end, I implemented a simple type of imperfect information within a standard sovereign default model and showed that it replicates the pattern of excessive optimism prior to both crises and defaults and, to varying extents, it can help close the gap between the perfect information model's predictions regarding debt accumulation prior to key events and the data.

Furthermore, the implementation of information imperfections in this model results in agents that are not particularly poorly informed – learning is quite quick and in general very accurate (at their worst point prior to a default, the average level of bias does not even exceed one standard deviation of the persistent process's innovation variance). However, those small differences in information structure result in equilibrium behavior that produce patterns of debt accumulation prior to key events that substantially better resemble the data. More extreme assumptions about the information structure could easily result in substantially larger differences in outcomes.

Of course, information imperfections are unlikely to be the sole element that explains the behavior of Greece (and other peripheral European countries) during the crisis of 2008−2012, but Paluszynski (2019) provides strong evidence for their existence, and I show here that incorporating it into a model can go help the model reproduce the relevant patterns of debt accumulation prior to key events. That said, the presence of the EU (and therefore the higher potential of a bailout), the ECB's purchases of assets, the possibility of renegotiation, and the ongoing (at the time) banking crisis are all not elements of the model surely contributed to the behavior of these governments. Indeed, each of these is certainly most relevant in the final few quarters of the time series, exactly those quarters which the model-generated data does not match the true series particularly well.

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6 Appendix

6.1 Appendix 1 - Detrending the Problem

Under repayment the government's problem is:

$$W^R(\Gamma_-, G_-, s, a) = max_{c,a'}\ u(c) + \beta E[W(\Gamma, G, s', a')|\Gamma] \tag{16}$$

such that

$$c + q(\Gamma, G, a')(a' - (1-\lambda)a) \leq G * (x+m) + (\lambda + (1-\lambda)\kappa)a \tag{17}$$

$$\Gamma = T(\Gamma_-, g, x) \tag{18}$$

$$G = g * G_- \tag{19}$$

Under default, the government's value is given by:

$$\begin{aligned} W^D(\Gamma_-, G_-, s) &= u(G * (x+m) - G * \phi(g, x)) \\ &\quad + \beta E[\theta W(\Gamma, G, s', 0) + (1-\theta) W^D(\Gamma, G, s')|\Gamma] \end{aligned} \tag{20}$$

$$\Gamma = T(\Gamma_-, g, x) \tag{21}$$

$$G = g * G_- \tag{22}$$

The government's problem at the beginning of the period when not in default is then:

$$W(\Gamma_-, G_-, s, a) = max_{d\in\{0,1\}}(1-d)W^R(\Gamma_-, G_-, s, a) + dW^D(\Gamma_-, G_-, s) \tag{23}$$

Now guess that the value functions are homogeneous of degree $1-\gamma$ in G and a and the price function is homogeneous of degree 0 in G and a'. Set $\hat{a} = \frac{a}{G}$, $\hat{a'} = \frac{a'}{G}$, $\hat{c} = \frac{c}{G}$ and verify first the budget constraint:

$$G * \hat{c} + q(\Gamma, 1, \hat{a}')(G * \hat{a}' - (1-\lambda)G * \hat{a}) \leq G(x+m) + (\lambda + (1-\lambda)\kappa)G * \hat{a}$$

$$\hat{c} + q(\Gamma, 1, \hat{a}')(\hat{a}' - (1-\lambda)\hat{a}) \leq (x+m) + (\lambda + (1-\lambda)\kappa)\hat{a}$$

and then the value function under repayment:

$$G^{1-\gamma}W^R(\Gamma_-, 1, s, \hat{a}) = G^{1-\gamma}u(\hat{c}) + \beta E[G'^{1-\gamma}W(\Gamma, 1, s', \frac{\hat{a}'}{g'})|\Gamma]$$
$$W^R(\Gamma_-, 1, s, \hat{a}) = u(\hat{c}) + \beta E[g'^{1-\gamma}W(\Gamma, 1, s', \frac{\hat{a}'}{g'})|\Gamma]$$

and then the value function under default:

$$\begin{aligned} G^{1-\gamma}W^D(\Gamma_-, 1, s, m) &= G^{1-\gamma}u((x+m) - \phi(g, x)) \\ &\quad + \beta E[\theta G'^{1-\gamma}W(\Gamma, 1, s', 0) + (1-\theta)G'^{1-\gamma}W^D(\Gamma, 1, s')|\Gamma] \\ W^D(\Gamma_-, 1, s, m) &= u((x+m) - \phi(g, x)) \\ &\quad + \beta E[\theta g'^{1-\gamma}W(\Gamma, 1, s', 0) + (1-\theta)g'^{1-\gamma}W^D(\Gamma, 1, s')|\Gamma] \end{aligned}$$

Note that no part of the problem now depends explicitly on G or t (and none of the transition probabilities for the remaining variables do either). Thus the detrended problem presented in the body of the paper is equivalent to the original problem described here.

6.2 Appendix 2 - Discretizing Beliefs

The vast majority of models involving beliefs in economics have, for computational purposes, used the prior distribution and the current signal as their state variables. When using discrete state space methods, this results in posterior distribution parameters falling in between grid points. At this time, only one paper, Fitzgerald et al. (2017), has used an implementation of belief updating with discrete state space methods which keeps the state variables on a grid and therefore does not require interpolation.

In other applications, it is reasonable to assume that the functions involved are differentiable up to some order and use interpolation to approximate the values off the grid. In the case of sovereign default models, it is common to suspect–or even to know–that the functions in question have points of nondifferentiability due to the nature of the discrete choice of whether to default. Furthermore, these kinks occur at extremely important points in the state space (including the boundary between the default set and the repayment set). For this reason (as well as problems ensuring convergence), a large part of the sovereign default literature has avoided using interpolation-based methods. Below, I describe a method for modelling the evolution of beliefs about a single $AR(1)$ process while keeping the distribution state variable on a grid.

Specifically, for $z = z_i, \epsilon = \epsilon_i, \eta = \eta_i, \rho = \rho_i, \mu = \mu_i$ with $i \in \{g, x\}$ with $\eta \sim \mathcal{N}(0, \sigma_\eta^2)$ and $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$, suppose the process z evolves by:

$$z_{t+1} = (1-\rho)\mu + \rho z_t + \eta_{t+1} \tag{24}$$

but the sequence

$$y_t = z_t + \epsilon_t \tag{25}$$

is observed. Given any initial normal prior $\mathcal{N}(\bar{z}_0, \sigma_{z,0}^2)$ on the time 0 value, the posterior variance evolves deterministically following:

$$\frac{1}{\sigma_{z,t}^2} = \frac{1}{\rho^2 \sigma_{z,t-1}^2 + \sigma_\eta^2} + \frac{1}{\sigma_\epsilon^2} \tag{26}$$

And the posterior mean evolves following:

$$\bar{z}_t = \frac{\sigma_{z,t}^2}{\rho^2 \sigma_{z,t-1}^2 + \sigma_\eta^2}(\rho \bar{z}_{t-1} + (1-\rho)\mu) + \frac{\sigma_{z,t}^2}{\sigma_\epsilon^2} y_t \tag{27}$$

Or, equivalently:

$$\bar{z}_t = (\rho \bar{z}_{t-1} + (1-\rho)\mu) + \frac{\sigma_{z,t}^2}{\sigma_\epsilon^2}(y_t - (\rho \bar{z}_{t-1} + (1-\rho)\mu)) \tag{28}$$

If we assume that the process has been observed for a very long time and the posterior variance has converged to the unique strictly positive fixed point of (26), then this becomes:

$$\bar{z}_t = (1-\rho)\mu + \rho \bar{z}_{t-1} + \frac{\sigma_z^2}{\sigma_\epsilon^2}(y_t - (\rho \bar{z}_{t-1} + (1-\rho)\mu)) \tag{29}$$

One property of the filter implied by Bayesian updating in the case of a normal prior with normal noise is that the forecast errors are mean zero independent normal random variables.

Stationarity of the posterior variance would imply that they are also identically distributed. Therefore (29) describes an $AR(1)$ process. This beliefs process can easily be discretized using standard methods. Upon a transition, the signal witnessed can then be set to exactly satisfy (29), i.e. by:

$$y(\bar{z}, \bar{z}_-) = \frac{\sigma_\epsilon^2}{\sigma_z^2}\left(\bar{z} - \frac{\sigma_z^2}{\rho^2\sigma_z^2 + \sigma_\eta^2}(\rho\bar{z}_- + (1-\rho)\mu)\right) \tag{30}$$

Standard discretizations are usually tested for their accuracy by comparing simulated moment values (ex. mean and variance) to their theoretical values. The implementation described above produces two related random variables, however, not just one. To determine its accuracy at different levels of fineness, I compute the full set of first and second moments for each variable as well as the forecast error and true error and compare them to their theoretical values. As can be seen below for some key quantities, using 100 points provides a highly accurate discretization. The parameters used for these simulations are the estimated income process parameters described in table 1.

Table 7: Discretized Belief Moments

Moment	**Theoretical**	**N=10**	**N=25**	**N=50**	**N=100**	**N=200**
$\sigma(z-\bar{z})$	0.013	0.015	0.014	0.013	0.013	0.013
$\sigma(y_t - \rho\bar{z}_{t-1})$	0.025	0.031	0.027	0.026	0.026	0.025
$\sigma(\bar{z})$	0.060	0.071	0.062	0.059	0.060	0.060
$\sigma(z)$	0.061	0.073	0.063	0.061	0.061	0.061
$\sigma(y)$	0.063	0.076	0.065	0.063	0.063	0.063
$\sigma(\epsilon)$	0.017	0.021	0.019	0.017	0.017	0.017
$\rho(z-\bar{z})$	0.428	0.397	0.428	0.428	0.428	0.428
$\rho(y_t - \rho\bar{z}_{t-1})$	0	-0.042	-0.018	-0.006	-0.002	$\lvert.\rvert < 1e-3$
$\rho(\bar{z})$	0.971	0.970	0.969	0.970	0.971	0.971
$\rho(z)$	0.971	0.975	0.971	0.971	0.971	0.971
$\rho(y)$	0.903	0.896	0.893	0.898	0.902	0.903
$\rho(\epsilon)$	0	-0.032	$\lvert.\rvert < 1e-3$	$\lvert.\rvert < 1e-3$	$\lvert.\rvert < 1e-3$	$\lvert.\rvert < 1e-3$

Table 8: Discretized Belief Moments Relative To True Values

Moment	**N=10**	**N=25**	**N=50**	**N=100**	**N=200**
$\sigma(z-\bar{z})$	1.159	1.082	1.020	1.007	1.002
$\sigma(y_t-\rho\bar{z}_{t-1})$	1.223	1.077	1.018	1.006	1.002
$\sigma(\bar{z})$	1.200	1.029	0.996	1.0	1.0
$\sigma(z)$	1.196	1.028	0.996	1.0	1.0
$\sigma(y)$	1.202	1.033	0.997	1.0	1.0
$\sigma(\epsilon)$	1.238	1.010	1.024	1.008	1.003
$\rho(z-\bar{z})$	0.928	0.999	0.999	1.0	1.0
$\rho(\bar{z})$	0.999	0.997	0.999	1.0	1.0
$\rho(z)$	1.003	0.999	0.999	1.0	1.0
$\rho(y)$	0.993	0.989	0.995	1.0	1.0

6.3 Appendix 3 - Haircut Calculations

Data from Zettelmeyer et al. (2013) and Trebesch and Zettelmeyer (2018) allow the calculation of the following terms for each bond included in the 2012 restructuring. The following procedure, based on the method described by Sturzenegger and Zettelmeyer (2008), was used to calculate the portfolio of defaulted obligations:

1. Calculate a risk free discounted unit value for each bond.

$$P_i^{rf} = e^{-rt_{i,N_i}} + \sum_{k\in 1:N_i} e^{-rt_{i,k}} * \kappa_i \tag{31}$$

2. Calculate the risk free discounted value of one unit of the EFSF contribution.

$$P^{EU} = \sum_{k\in 1:N_{EU}} e^{-rt_{EU,k}} * (\pi_{EU,k,k} + \kappa_{EU,k}\pi_{EU,k,N_{EU}}) \tag{32}$$

   where $\pi_{EU,k,l}$ is the fraction of $EFSF$ notes maturing between times $t_{EU,k}$ and $t_{EU,l}$, inclusive.

3. Calculate a single risk free discounted unit value for the bond portfolio received by investors participating in the exchange.

$$P^{EX} = \sum_{k\in 1:N_{EX}} e^{-rt_{EX,k}} * (\pi_{EX,k,k} + \kappa_{EX,k}\pi_{EX,k,N_{EX}}) \tag{33}$$

4. Calculate the unit value defaulted on as:

$$D_i = P_i^{rf} - 0.315 * P^{EX} - 0.15 * P^{EU} \tag{34}$$

5. Calculate the original unit equivalent of the defaulted portion of each bond issue as:

$$\hat{D}_i = \frac{D_i}{P_i^{rf}} \tag{35}$$

6. Multiplying $\hat{D}_i$ by the face value of bond issue i then yields the effective face value of the defaulted portion of that bond issue (where defaulted now implies a 100% haircut):

$$\hat{B}_i = \hat{D}_i * B_i \tag{36}$$

7. We may also determine the weighted average original unit haircut as:

$$\bar{D} = \frac{\sum_{i=1}^{N_B} \hat{D}_i * B_i}{\sum_{i=1}^{N_B} B_i} \tag{37}$$

This whole procedure results in a $\bar{D}$ of about 63% (which should not be surprising for an official face value haircut of 53.5% which dramatically extended the maturity of the remaining debt and provided almost a third of it at risk free prices).

Since Greece's debts to the ECB, National Central Banks, and the EIB (as well as a small number of holdouts) were not restructured, I multiply the resulting haircut by the ratio of the total nominal value participating to the total nominal value outstanding (about 75%). This yields a final effective haircut haircut of just over 47.5%.