Liquidity, Default Risk, and the Information Sensitivity of Sovereign Debt

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(Preliminary)

Abstract

In this paper, I document that, during the height of the Eurozone Debt Crisis in Spain, 1.) Spanish government bonds became substantially less liquid and less traded on secondary markets, 2.) the first appearance of this phenomenon lagged far behind the initial jump in interest rate spreads in late 2008, and 3.) it persisted throughout the period of peak interest rate spreads and only subsided after the worst of the crisis had passed. I argue that these facts are related and best explained by a model of sovereign default that features secondary markets in which it is possible that some traders have private information. I then build a model in which some traders have private information about the country's future economic conditions and show that this allows the model to reproduce both the delayed reaction of bid-ask spreads as well as their peak and behavior during the height of the crisis. Using the model, I measure the losses to investors associated with variation in liquidity during debt crises. Finally, I validate the model by showing that the model's predicted relationship between current, realized bid-ask spreads and future values of GDP allows me to forecast GDP significantly better than a standard, benchmark forecast.

1 Introduction

The Eurozone debt crises of $2008-2014$ were an unprecedented event for the modern, developed world. One country (Greece) was forced to default, three more (Ireland, Portugal, and Cyprus) had to be rescued with bailouts, and another two (Spain and Italy) required more general support to avoid disaster. Until this time, sovereign debt crises were thought to be primarily a problem of Emerging Market Economies and Developing Countries. While the cases of Portugal and especially Greece are somewhat similar in certain ways to debt crises in Emerging Market Economies that have been studied at length[1], the cases of Spain, Italy, and Ireland had no such parallel. Spain and Ireland had relatively low levels of government debt prior to the crisis, and Italy had been steadily reducing its debts over the decade prior to the crisis. Furthermore, secondary market activity played a significantly larger role in the evolution of the crisis overall.

This paper focuses on documenting and rationalizing that secondary market activity in the experience of Spain. As countries came under stress, the markets for their outstanding debts began to break down. This is not necessarily a natural outcome. Certainly, prices should fall as default risk rises, but that does not imply that it should become harder to find someone willing to trade at those prices. However, the liquidity of each country's debt declined sharply during the peak of their crisis, and those debts become significantly harder to trade.

In this paper, I study the relationship between choices of government borrowing and the liquidity of outstanding debt. I document that the liquidity of Spanish bonds during the Eurozone debt crises, as measured by bid-ask spreads in secondary markets, features an highly nonlinear relationship with interest rate spreads and implied default risk. Specifically, bid-ask spreads barely reacted at all to the first half (measured in time or in magnitude) of the rises in interest rates. Once interest rates passed a certain threshold, bid-ask spreads jumped from their prior levels and began to comove closely with interest rate spreads. I argue that this pattern is best explained by the presence of private information, obtained at a

[1] Both ran large current account deficits and issued debt at a rapid pace in order to fund a consumption binge during the relatively good economic times of $2001-2008$. When circumstances soured in late 2008 and early 2009, before deteriorating much further over the next few years, both were caught with huge stocks of debt that were unsustainable as their economies contracted rapidly.

cost. I then build a quantitative model of sovereign default incorporating frictional secondary markets in which some agents have access to private information to show that this mechanism can induce produce the observed patterns. I calibrate the model to match the experience of Spain during the Eurozone debt crises. I then examine the model's predictions of how crises should evolve and show they match the experience of Spain quite well. Furthermore, in this model, the private information that some investors obtain pertains to the country's future GDP values, which induces a relationship between current, realized bid-ask spreads, and future output levels. In order to validate the model, I confirm that using this relationship allows me to better predict the evolution of output during the Eurozone debt crises than a benchmark forecast would.

The three most common traditional explanations for the existence of bid-ask spreads are:

1. the presence of private information (or even just the possibility of its presence);
2. transaction costs charged by intermediaries (or included as a markup to prices by market makers);
3. inventory risk (especially for risk averse intermediaries).

The first is the most plausible explanation for the patterns in the data. The presence of private information in a market can easily generate substantial differences between posted bid and ask prices. Furthermore, if said information is acquired at cost, then rates of acquisition will vary with the value of the information. When there is very little dispersion in paths for the future (or at least, very little dispersion which can be distinguished using the information), the information is less valuable. As dispersion in possible futures rises, the information becomes more valuable. Therefore, default risk can rise without immediately triggering problems in secondary markets. When default risk rises enough, however, liquidity changes sharply as some agents pay to acquire information and other agents react to that choice.

Transaction costs are not, in general, a value which one would expect to vary significantly over time, or covary with features of the business cycle or measures of default risk. As for inventory risk, most standard models applying it would predict has significantly less nonlinear relationship between future expected risks and liquidity than is observed in the data.

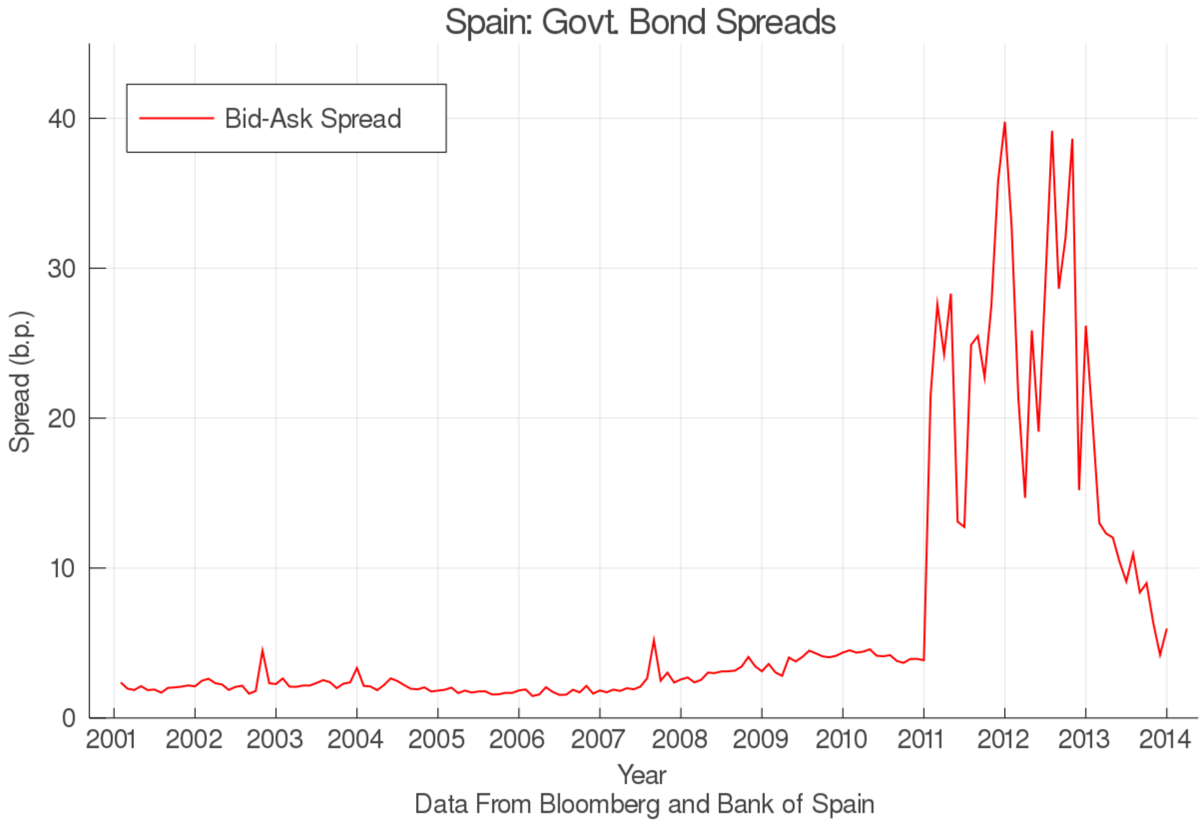
Furthermore, throughout $2001 - 2016$, Spanish government debt constituted one of the ten biggest single markets in the world. During that time, the average outstanding face value of its debt securities averaged just over 500 billion Euros. The average turnover in secondary markets as a percent of units outstanding over the same period was 0.8% daily (or 15.4% monthly). Even during the worst stages of the crisis in $2011-2013$, the 1^{st} and 5^{th} percentiles for turnover were, respectively, 0.06% and 0.14% daily (or 4.5% and 5.6% monthly). Just the daily numbers represent total exchanges of $300 - 700$ million euros worth of face value. I point this out simply to note that the even observed turnover rates far imply substantial activity. This was never a thin market, and only the very largest intermediaries could not have offloaded their entire holdings of Spanish debt within the worst month.

2 Data

The most commonly used proxy for the liquidity of an asset is the bid-ask spread: the difference between exchange-posted prices for an immediate sale and an immediate purchase. The bid price is the highest price attached to an active, unfilled buy order submitted to the exchange, and is therefore, to an investor holding the asset, the price for an immediate sale. The ask price is the lowest price attached to an active, unfilled sell order submitted to the exchange, and is therefore, to an investor holding the asset, the price for an immediate purchase. The difference between the two must therefore be positive, and is known as the bid-ask spread. When bid-ask spreads are narrow, perhaps accounted for fully by transaction costs, almost all trades for which a real surplus exists occur. For any given distributions of seller valuations and potential buyer valuations, if observed bid-ask spreads widen, it must be the case that relatively more trades for which a real surplus exists do not occur. At least one potential participant in each such trade is choosing not to trade by bidding below their true valuation or asking above it. Markets and circumstances which lead to more and larger potential surpluses from trade not being realized are often termed "illiquid," which is one reason why bid-ask spreads are a good proxy for liquidity.

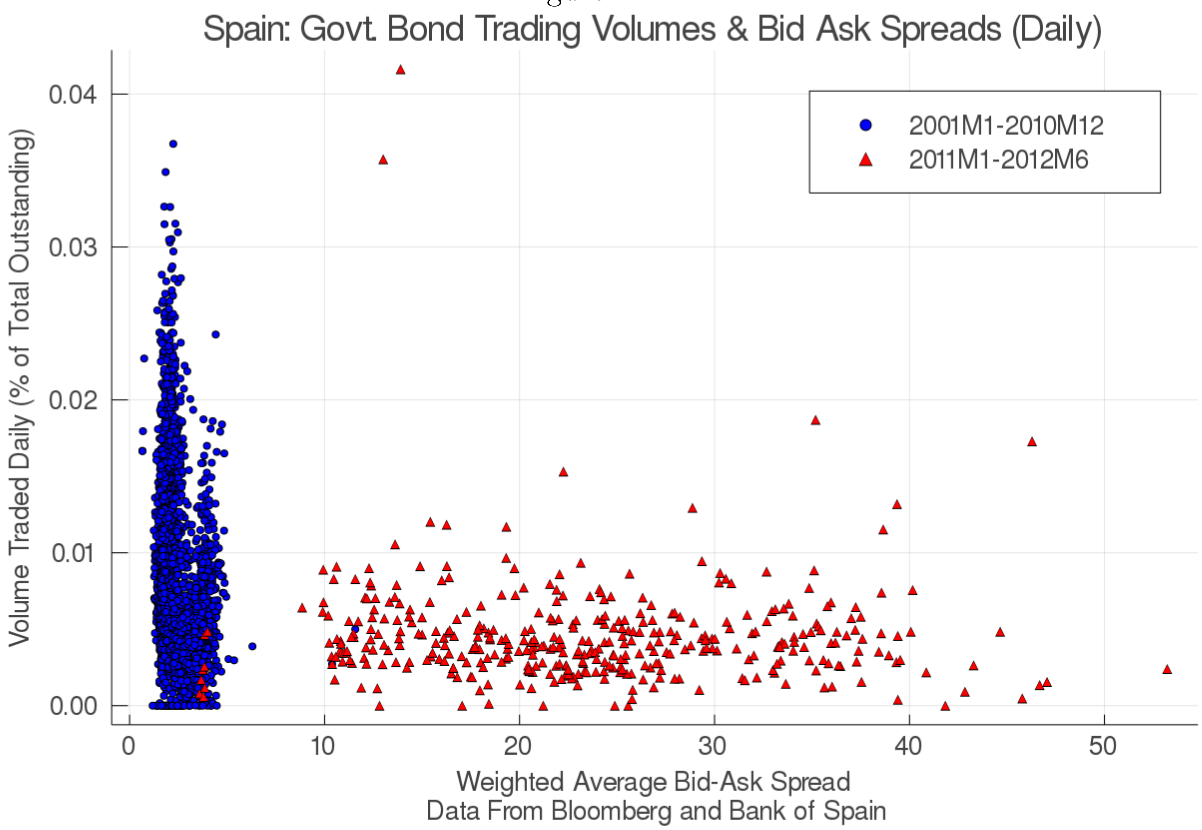
During the height of the Spanish debt crisis, the liquidity of Spanish government bonds changed sharply. Figure 1 plots the monthly weighted average difference between interest rates demanded by buyers of Spanish government bonds and those demanded by sellers:

Figure 1:



The beginning of the period of significantly higher than average levels is January 2011. To confirm that the period from then until June 2012 (the last month before Mario Draghi made his "Whatever It Takes" speech) is associated with markedly subdued activity in secondary markets, I plot in figure 2 daily weighted average bid-ask spreads against the average daily trading volumes in secondary markets (measured as total number of units exchanged divided by total number of units outstanding).

Figure 2:



The purpose of this picture is to show that when bid-ask spreads are elevated, the range of possible outcomes for volumes narrows substantially. Furthermore, while 20-50 basis points might be particularly large relative to average interest rate spreads in emerging economies, these numbers come from a country which paid average interest rate spreads of just 0.35% over the prior 10 years and an average of just 0.72% over the full period 2001M1-2012M6.

In any case, were it not for the fact that essentially all of the data points associated with high bid-ask spreads occurred in certain relatively narrow time period, the above plot would only be interesting insofar as it shows why bid-ask spreads are a useful proxy for liquidity. However, this exact time period coincides exactly with the section of the crisis during which interest rate spreads on Spanish government debt were highest. Figure 3 plots the spread between benchmark Spanish government bonds and their German counterparts.

Figure 3:

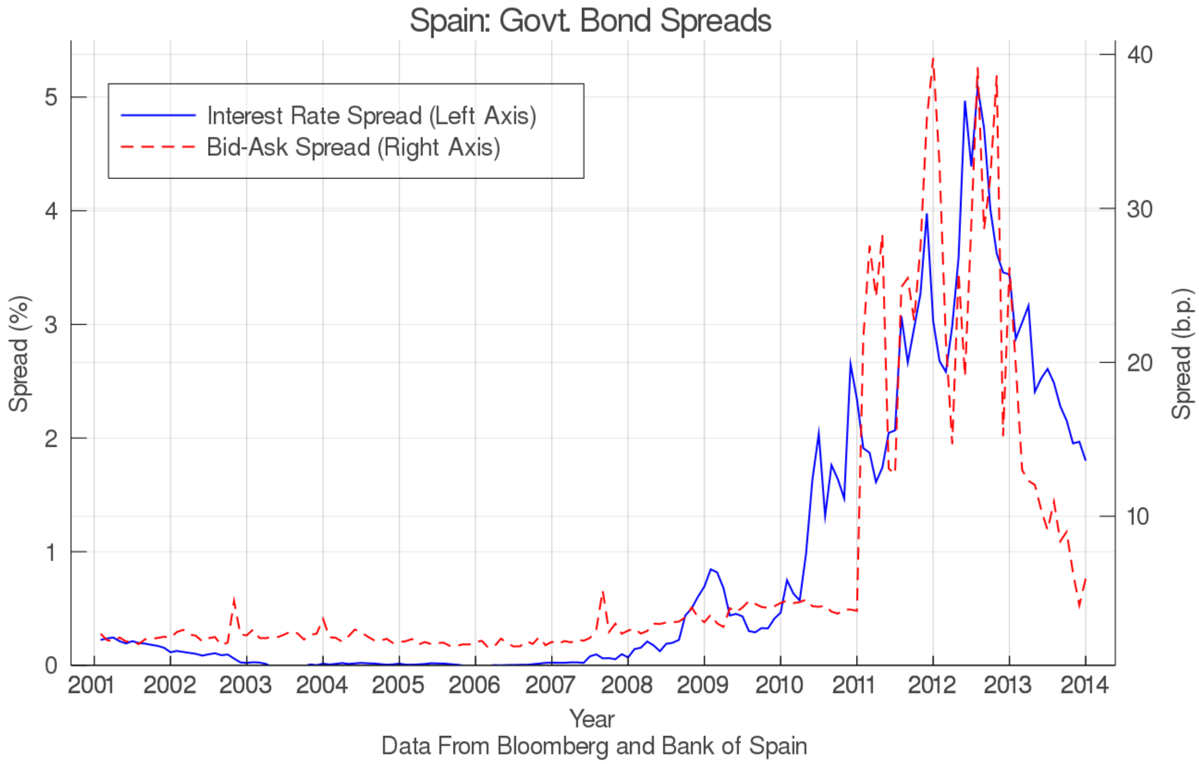
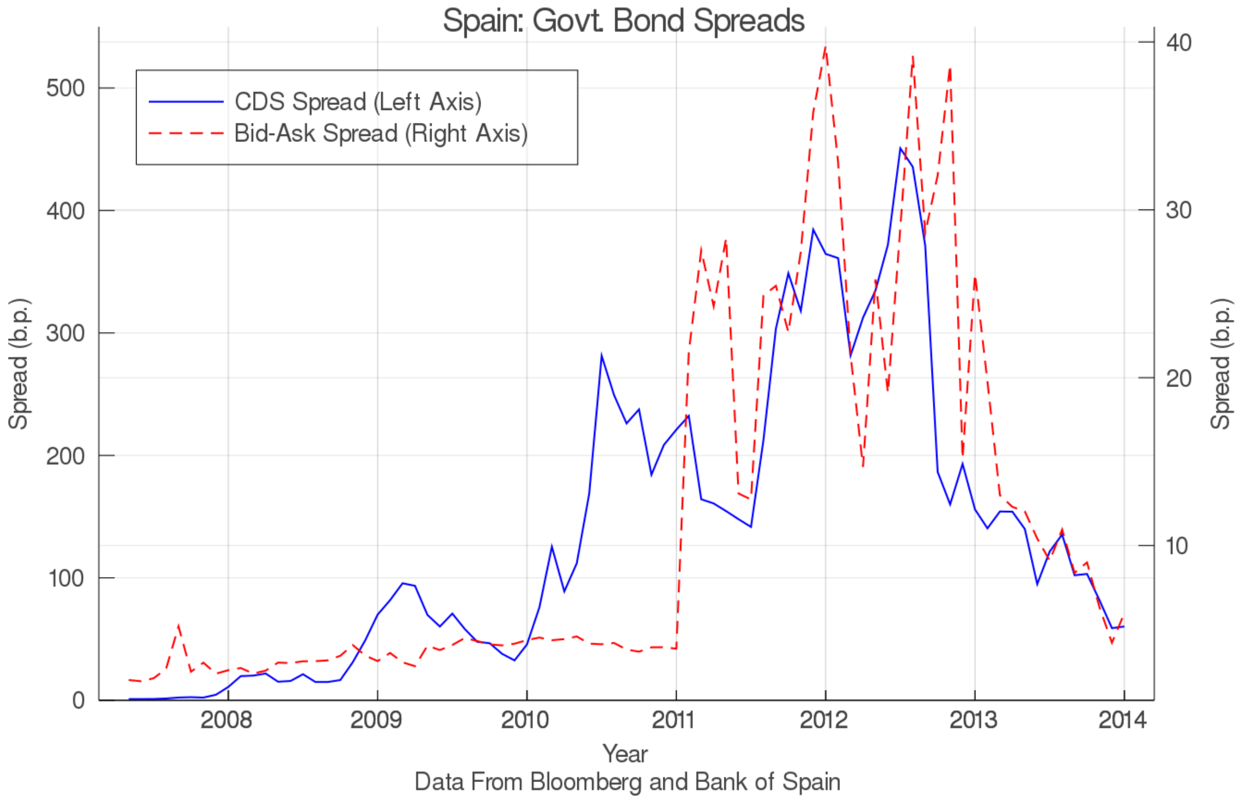


Figure 3 shows that in the three years from 2008 to 2011, interest rate spreads rose from essentially 0 to about 2%, with barely any reaction at all from bid-ask spreads. As they continued to rise during 2011, suddenly bid-ask spreads began responding, and jumped significantly away from their prior low levels. In order to tease apart the effects of expected future liquidity risks from those of default risk itself, I turn to the probability of default over various horizons implied by the price of credit default swaps. Figure 4 plots the monthly average 1 year CDS spread for contracts on Spanish government debt from 2007 to 2014:

Figure 4:



As before, the period of unusually high bid ask spreads also corresponds to the period during which CDS spreads peak. Again, however, the CDS spreads began to rise much earlier, approximately in line with the interest rate spread series. Furthermore, the implied short term default risk is only higher than its value in mid 2010 for the middle of the period.

3 Literature Review

The papers in the sovereign default literature most closely related to this one are Passadore and Xu (2018) and Chaumont (2018). Both incorporate frictional secondary markets into otherwise standard model of sovereign defaults in order to produce endogenous bid ask spreads which covary with interest rates and default risk. Passadore and Xu (2018) study the case of Argentina's 2001 default. Their model uses investors of two types (high and low, with high types randomly transitioning to low types), the lower of which has less bargaining power when their bondholdings are in default, to produce bid ask spreads which widen as default risk increases.

Chaumont (2018) studies the case of Greece's 2012 default. The basic mechanism which produces the differences in valuations which lead to gains from trade is identical to that of Passadore and Xu (2018) (high and low type investors who vary by their preference for owning the government's bond). However, he also explicitly models the primary and secondary markets separately and distinguishes between primary dealers and normal investors. Investors and dealers engage in a directed search problem to choose which market to enter. Markets are distinguished by the transaction fees associated with trading in them. These transaction fees are the gross revenues of dealers, and their profits net out a constant cost of participation in any submarket. Bid ask spreads are driven entirely by the equilibrium distribution of transaction fees. They covary with interest rates and default risk because, as default risk rises, the low type investors begin to panic and choose to participate in markets with higher transaction costs in order to quickly sell their bonds. This changes market tightness across submarkets, and dealers and high type investors also adjust so that markets clear, but the equilibrium outcome is higher average transaction fees. In my model, bondholders never have persistently different preferences. Furthermore, I do not directly model intermediaries, but instead develop a trading protocol that allows me to easily characterize behavior when one party has an informational advantage over the other.

Another pair of related papers are Cole et al. (2020) and Cole et al. (2021). Both consider the effects of information asymmetry among investors on outcomes in the primary market. The model of Cole et al. (2020) is more stylized and designed to illustrate a mechanism by which asymmetric information about default risk can impact interest rates paid by the government when auctioning debt. Motivated by data on Mexican government debt auctions, Cole et al. (2021) is a quantitative paper more in line with mine. The authors show that bids from larger bidders are significantly more likely to be accepted, and that this observation is consistent only with the presence of asymmetric information among investors. After calibrating the model to match the specific patterns they observe in the data, they find that information asymmetries can have economically significant effects on the yields paid by the government when it auctions new debt. While my model also incorporates information asymmetries among investors, those frictions affect behavior in secondary markets, rather than in primary markets. In primary markets, my model features symmetric information among all participants.

This paper also connects to the broader literature on the information sensitivity of debt. Gorton and Ordoñez (2014), Gorton and Ordoñez (2019), and Dang et al. (2015) all consider environments with interconnected choices of borrowing and information acquisition. A common theme in those papers is that in normal times, when average levels of risk are lower, a relatively large set of borrowing values does not trigger information acquisition. On the other hand, in periods of higher risk, the information insensitive set of borrowing values shrinks, more borrowers make choices which end up triggering information acquisition, and equilibrium prices must "compensate" creditors for the cost of acquiring that information. This paper will feature a broadly similar dynamic, albeit in a relatively simple secondary market.

More specifically, the key ingredients will be:

1. a secondary market with anonymous trading;
2. random differences in fundamental valuations of government bonds between buyers and sellers;
3. the ability of some (or all) agents to acquire private information which is relevant to the payoff of holding the bond;

Since in general (regardless of how the surplus from trade is shared), trades will clear when a buyer offers a price higher than the seller's valuation, all three will be important. The reason for 3. is obvious. However, without 2., if no agent acquires information, the outcome is indeterminate, and no trade theorems guarantee that, if any agent acquires information, no transactions clear. 1. guarantees that each side can never map exactly the other side's action to their own value. When there is a possible surplus as well as the possibility that the counterparty has more information than the agent, the agent cannot be sure whether the transaction is clearing because there actually is a real surplus or because the counterparty is exploiting his informational advantage.

4 Model

Time is discrete and infinite. There is a small open economy with a representative consumer and a benevolent government who have identical recursive preferences over consumption and continuation values given by:

$$V = U(c, V') \tag{1}$$

$U : \mathbb{R}_{++} \times \mathbb{R}_{++} \to \mathbb{R}_{++}$ is a continuously differentiable function which is strictly increasing, homogeneous of degree 1, and satisfies $\lim_{c->0} U_1(c, V') = +\infty$ and $\lim_{V'->0} U_2(c, V') = +\infty$. The exogenous stochastic state of the world s is a Markov Process and governs country's GDP $y(s)$.

The government may borrow on international markets using a defaultable long term bond. Following Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), I use a probabilistic characterization of maturity. Specifically, each bond matures with constant probability λ each period. With complementary probability, the bond instead pays a coupon κ. There is a continuum $[0, \bar{B}]$ of risk-neutral, competitive international lenders, each of whom can hold a unit of the bond. When it defaults, the country enters financial autarky and suffers an output penalty $\phi(s)$. It exits autarky at constant rate θ.

When in good standing, the government's problem, at the beginning of the period is:

$$W(s, b) = max_{d \in \{0,1\}} (1 - d) W^R(s, b) + d W^D(s) \tag{2}$$

Conditional on repayment, its problem is:

$$W^R(s, b) = \max_{c, b',} U(c, \bar{W}(s, b')) \tag{3}$$

such that

$$c + (\lambda + (1 - \lambda)\kappa) b = y(s) + q(s, b')(b' - (1 - \lambda) b) \tag{4}$$

where $\mu(.)$ is a certainty equivalent operator and the value of exiting the period not in default is given by:

$$\bar{W}(s, b') = \mu(W(s', b')|s) \tag{5}$$

Conditional on default, its value is:

$$W^D(s) = U(y(s) - \phi(s), \bar{W}^D(s)) \tag{6}$$

where the continuation value associated with exiting the period while in default $\bar{W}^D(s')$ is given by:

$$\bar{W}^D(s) = \mu(W(s', 0), W^D(s')|s) \tag{7}$$

The above problem simply the generalization of the one presented in Chatterjee and Eyigungor (2012) to allow for non-expected utility preferences. It differs only in the functional equation which the bond price $q(s, b')$ must satisfy. I now turn to describing the market structure in order to derive that equation.

Once the government has made its borrowing decision and the auction has been completed, there is a set $[0, b']$ of bondholders. Each of these agents has a random discount factor $\hat{\delta} \sim F(.)$. These discount factors are i.i.d. across agents and time, with $supp(F) = [\underline{\delta}, \bar{\delta}]$. They are not known at the beginning of the period. Once the auction is completed, a signal $\hat{s}'$ about the future exogenous state of the world s' is realized. Current investors may pay a cost $f(\pi)$ to attempt to learn the value of this signal. With probability π, they observe the signal, and with complementary probability, they learn nothing. After they have made that decision and observed the signal, their random discount factors are realized. This completes the set of events which occurs before the secondary market opens and trading begins.

I now turn to describing the trading protocol in the secondary market. In the secondary market, each current investor is matched with a new investor. All new investors have a fixed, known discount factor δ. New investors do not observe the discount factor of their match, or whether their match observed the signal $\hat{s}'$. Simultaneously, current investors submit an ask price and new investors submit a bid price. If the bid price exceeds the ask price, the transaction clears and the current investor is replaced by the new investor who bought their bondholdings.

Since investors are risk neutral and competitive (at the time of the initial auction), the price of the bond on the primary market must be exactly the expected value of going to the secondary market with a bond:

$$q(s,b') = \max_{\pi\in[0,1]} (1-\pi)q_U(s,b') + \pi q_I(s,b') - f(\pi) \tag{8}$$

where $q_U(.), q_I(.)$ denote the value of being uninformed or informed, respectively. $\pi^\star(s,b') \in (0,1)$ therefore implies:

$$q_I(s,b') - f'(\pi) = q_U(s,b') \tag{9}$$

Let $\pi_S(s,b')$ denote the equilibrium proportion of current investors which obtain access to s'.

5 Secondary Market Equilibrium

In order to derive the values associated with being informed or uninformed as well as the equilibrium proportion acquiring information, let us introduce some notation. Let v denote the undiscounted unit value of the asset to an uninformed agent.

$$v(s,b') = E[(1-d(s',b'))(\lambda + (1-\lambda)(\kappa + q(s', b''(s',b')))|s] \tag{10}$$

Let $\hat{v} \sim G(.)$ denote the random variable which is the undiscounted unit value of the asset to an informed agent (and of course $E[\hat{v}] = v$).

$$\hat{v}(s,\hat{s}',b') = E[(1-d(s',b'))(\lambda + (1-\lambda)(\kappa + q(s', b''(s',b'))))|s,\hat{s}'] \tag{11}$$

Now, the assumption that transactions clear at the bid price make truth telling a dominant strategy for sellers, so:

$$p^\star_{S,U}(\hat{\delta}) = \hat{\delta} v \qquad\qquad p^\star_{S,I}(\hat{\delta},\hat{v}) = \hat{\delta}\hat{v} \tag{12}$$

Knowing this, the problem solved by buyers is:

$$max_{p_B}(1-\pi_S)(\delta v - p_B)F(\frac{p_B}{v})$$
$$+\pi_S\Bigg(-Pr(\hat{v}=0)p_B + \int_V(\delta\hat{v} - p_B)F(\frac{p_B}{\hat{v}})dG(\hat{v})\Bigg) \tag{13}$$

Notice the difference between the terms multiplied by $(1-\pi_S)$ and π_S. In the first case, the integration has been moved all the way inside the expression to every term involving v. In the second case, the integration occurs outside, and the values of the resulting surpluses $\delta\hat{v}-p_B$ are multiplied by the probability $F(\frac{p_B}{\hat{v}})$ of meeting an agent with a discount factor low enough to accept the offer p_B. As $\hat{v}$ falls and the resulting surplus falls with it, the probability that the offer will be accepted rises, since fundamental valuations are correlated.

Therefore, if there is any dispersion in $\hat{v}$, the value to the buyer at any given bid is decreasing in the proportion of sellers which acquire information. This results in equilibrium average bid prices average for any fixed v which are decreasing as the dispersion of the $\hat{v}$ values rises, the value of acquiring information rises for any fixed bidding strategy, and more sellers acquire the information. However, the equilibrium average ask price is always v multiplied by the average discount factor of sellers. Since when default risk is high, information about the value of y' helps relatively more in discerning future values, bid prices drop while ask prices remain the same (in expectation), and the average bid-ask spread rises while the average volume of transactions falls. As the expected average bid price falls, however, the ex ante value to both uninformed as well as informed sellers of bond falls. Their reaction to this is to demand relatively higher interest rates when purchasing the bond from the government in the first place. This is the channel by which liquidity in the model flows from secondary markets back to the price schedule faced by the government at the price market.

In order to understand the way all of these forces interact, I walk through the determination of an equilibrium at an example point in the calibrated model. In the calibrated model, $\hat{\delta}$ has a uniform distribution and the cost of acquiring information is linear $f(\pi) = f * \pi$, so the seller's problem involves comparing q^U and $q^I - f$. I begin with a plot of these ex ante values for a range of bid prices:

Figure 5:

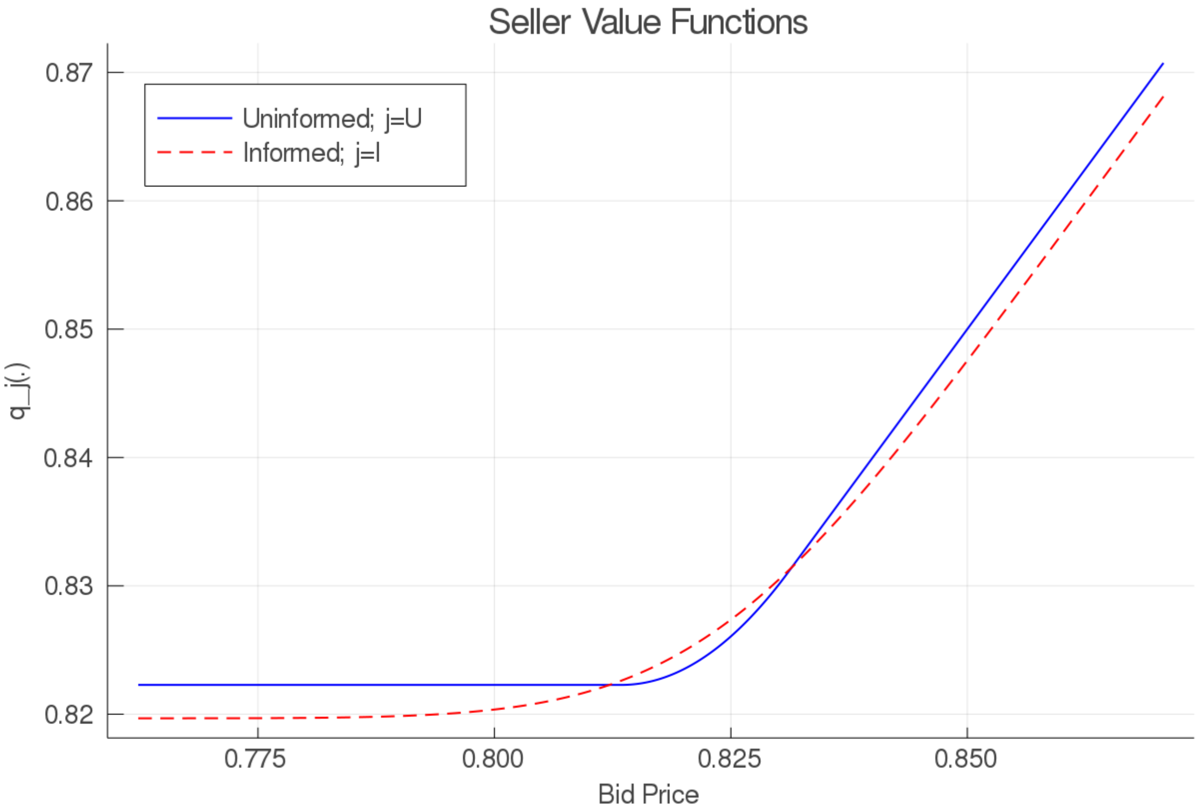
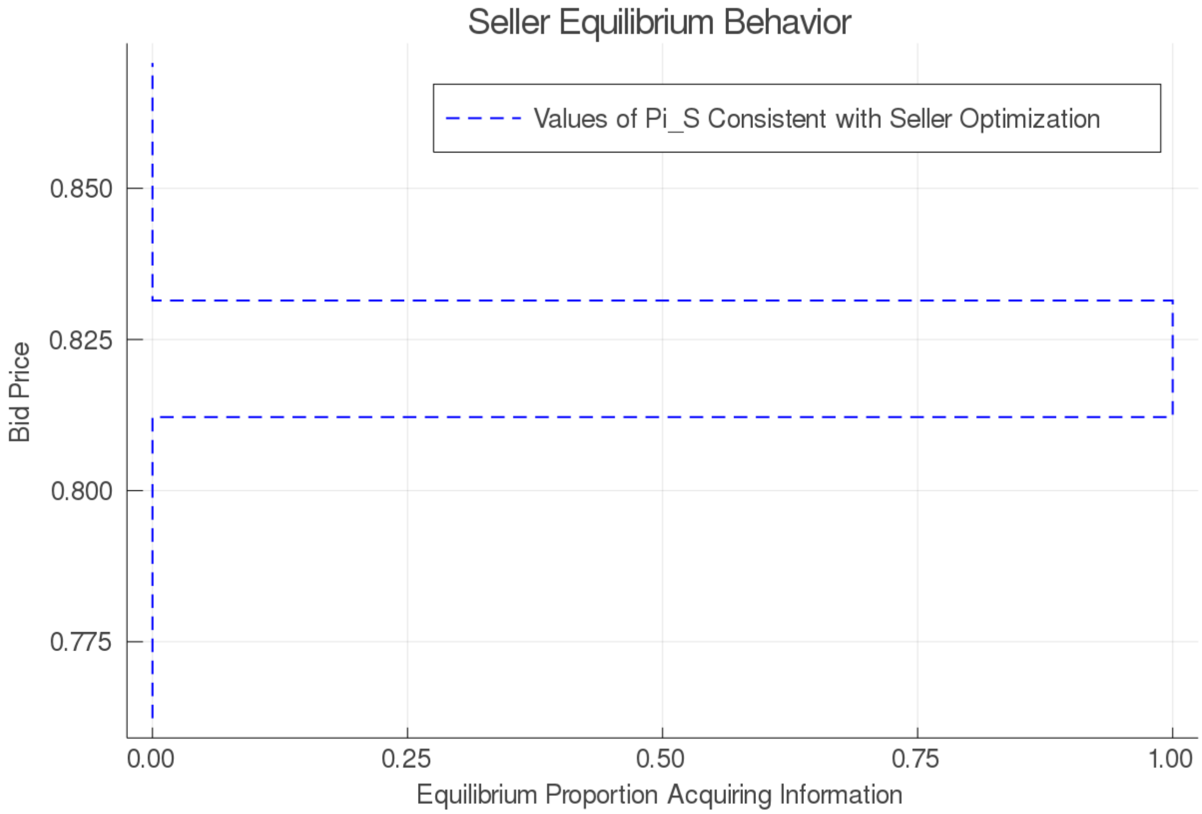


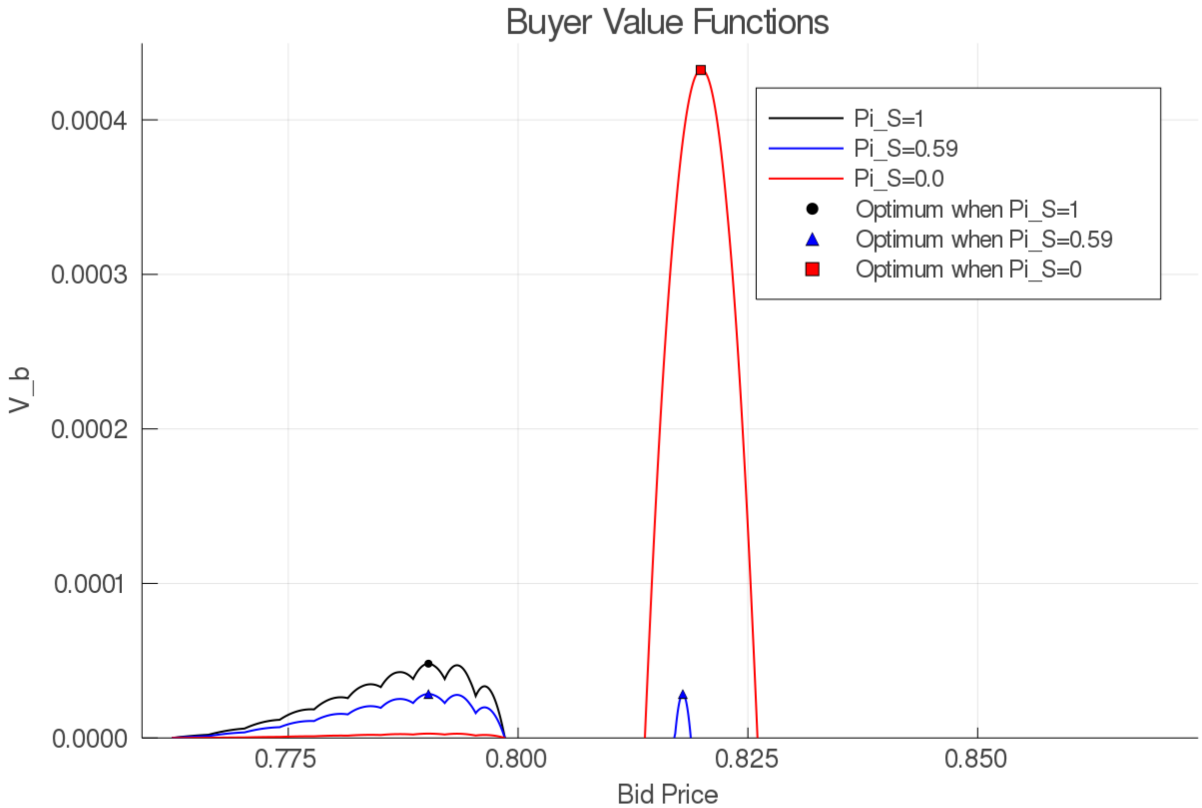
Figure 5 illustrates how information is not always worth the cost associated with acquiring it. In particular, when buyers bid very high values, sellers accept their offers almost regardless of the signal $\hat{s}'$, so knowing that signal does not allow them to make any extra profit. Similarly, when buyers bid very low values, sellers reject their offers similarly frequently, so observing $\hat{s}'$ again does not allow them to extract anything extra from buyers. For intermediate values, however, having access to $\hat{s}'$ is very useful and allows the seller to adjust their accept/reject decision quite often. In these cases, the sellers can use their information advantage to extract enough extra value from the buyers to more than cover the cost of acquiring the information in the first place. These patterns lead to the following best response plot for sellers:

Figure 6:



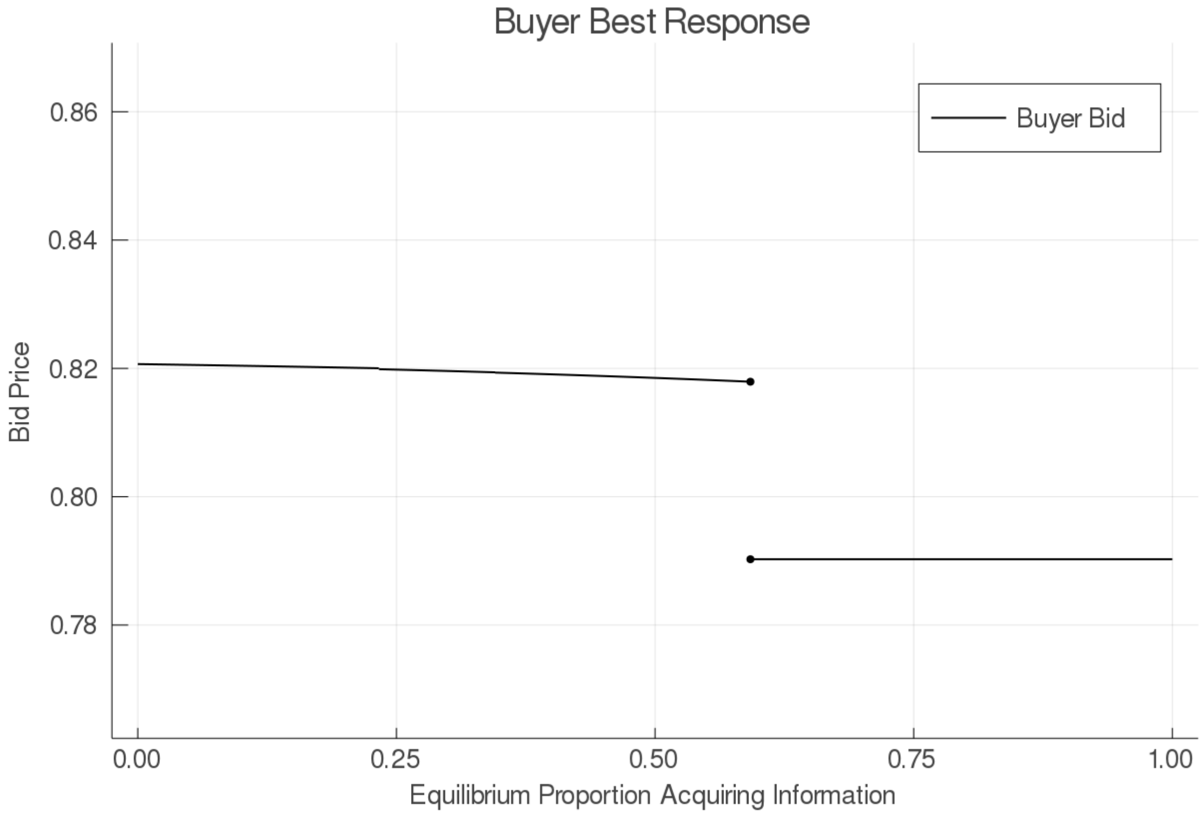
The levels at which the best response becomes flat correspond exactly to the points at which the sellers' values crossed in Figure 5. I now turn to the buyer's problem. Figure 7 below plots the value to the the buyer of various bids at a few specific values of π_S:

Figure 7:



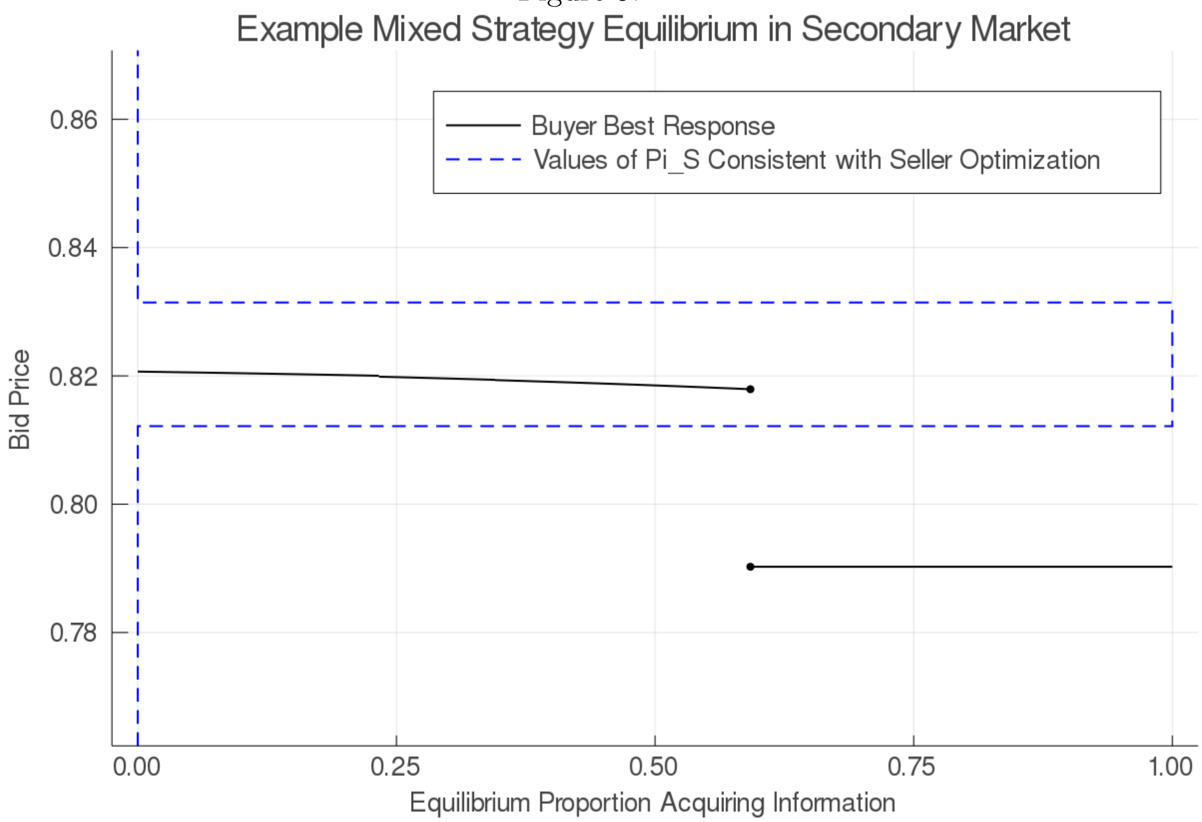
When no sellers acquire information, buyers are confident that they will be trading with uninformed sellers and submit relatively high bids. When all sellers acquire information, buyers are certain that they will be at a huge informational disadvantage. In order to protect themselves, they submit relatively low bids, and trade only occurs following relatively poor signals. Some trade does still occur, and buyers do gain some of the surplus, in expectation. At the intermediate value of π_S, there are two bid prices that yield the exact same value to the buyer. The lower of the two is exactly the same price buyers choose when 100% of sellers acquire information. This is because, by bidding so low, these buyers know they will never end up trading with uninformed sellers, so the only outcomes which concern them are meetings with informed sellers. When π_S takes this intermediate value, there is another bid price that allows buyers to trade with both uninformed sellers and informed sellers. This higher price exposes the these buyers to much heavier information rent extraction when they meet informed sellers, but the opportunity for larger gains from trade with uninformed traders just makes up for that. This leads to the following overall best response plot for buyers:

Figure 8:



If we overlay the two best response plots on top of one another, we obtain the following:

Figure 9:



In this case, there is a unique equilibrium, and it involves sellers choosing an intermediate value of $\pi^\star$ and some buyers playing a relatively high price while the remainder play a relatively low price. We can see based on the previous seller value functions that there exists some combination of buyer strategies that makes lenders indifferent among all possible choices of π, so the one which makes buyers indifferent between those two prices is an optimum. There are, in general, four characteristic types of equilibria in the secondary market. They are:

- No seller acquires information, buyers play identical pure strategies.
- All sellers acquire information, buyers play identical pure strategies.
- Only some sellers acquire information, buyers play identical pure strategies.
- Only some sellers acquire information, buyers mix between two pure strategies.

The equilibrium obtained in the above example was of type 4. In practice, all four types occur with nontrivial frequency in the calibrated model.

6 Calibration

The model is calibrated using data on the economy and government borrowing activities of Spain. GDP is assumed to be the sum of a persistent process y_t and an i.i.d. process m_t.[2] The persistent component of the income process y is parametrized as an $AR(1)$ process with normally distributed innovations:

$$y_t = \rho y_{t-1} + \eta_t \qquad\qquad \eta_t \sim N(0, \sigma_\eta^2) \tag{14}$$

The m process is parametrized as a symmetrically truncated normal random variable $m \sim TN(0, \sigma_m^2, -\bar{m}, \bar{m})$. The parameters $\rho, \sigma_\eta, \sigma_\mu$ are estimated using OECD data on Spanish Real GDP from 1986Q1 (the first quarter after Spain's accession to the European Union on January 1 1986) to 2012M6 using standard state space methods after the removal of a quadratic time trend. $\bar{m}$ is set to be $2 * \sigma_m$, a wide enough range to ensure convergence for a broad set of parameter values. Since liquidity is inherently a very short run issue, I estimate all parameters at monthly values and set the time step in the computational model to one month. The resulting estimates for the income process parameters are:

Table 1: Estimated Income Process Parameters (Monthly Values)

Parameter	**Value**	**SE**
ρ	0.9918	0.007
σ_η	0.0049	0.0005
σ_m	0.0015	0.0004
$\bar{m}$	0.0031	–

The maturity and coupon parameters of the asset structure are estimated using data on the individual Spanish bond issues outstanding in each month from 2001Q1 to 2012M6. The coupon parameter is set to be the weighted average coupon rate on outstanding debts

[2] The role of m_t is primarily to ensure convergence of the computational algorithm used to solve the model. See Chatterjee and Eyigungor (2012) for a more detailed explanation.

throughout the period. The maturity parameter is set to match the mean of the weighted average maturity of the country's outstanding debt obligations. Specifically, if at time t the country's debt obligations specify that at each time $\tau \in \mathscr{T}$ the country makes payments P_τ, then the weighted average maturity of that portfolio is:

$$\bar{M}_t = \frac{1}{\sum_{\tau \in \mathscr{T}} P_\tau} \sum_{\tau \in \mathscr{T}} (\tau - t) P_\tau \tag{15}$$

The utility function was set to be Epstein Zin:

$$U(c, V') = \begin{cases} \left((1-\beta)c^{1-\psi} + \beta V'^{1-\psi}\right)^{\frac{1}{1-\psi}} & \psi \neq 1 \\ c^{1-\beta} V'^{\beta} & \psi = 1 \end{cases} \tag{16}$$

$$\mu(V'(s')|s) = \begin{cases} \left(\int_{\mathscr{S}} V'(s')^{1-\gamma} dF(s'|s)\right)^{\frac{1}{1-\gamma}} & \gamma \neq 1 \\ exp\left(\int_{\mathscr{S}} ln(V'(s')) dF(s'|s)\right) & \gamma = 1 \end{cases} \tag{17}$$

The reentry parameter is set to the monthly equivalent of the quarterly 0.385 value used by Chatterjee and Eyigungor (2012). The distribution of $\hat{\delta}$ is set to be $U(\underline{\delta}, \bar{\delta})$. The values of $\bar{\delta}$, $\underline{\delta}$, and δ are fixed so that:

- the implied annualized risk free interest rate experienced by the government when $\pi_S = 0$ is 4%;
- the average bid-ask spread in secondary markets when $\pi_S = 0$ is 2.5 b.p.;
- the percent of outstanding debt traded every month is 37%.

The full set of non-income process parameters set outside the model is:

Table 2: Non-Income Parameters (Monthly Values)

Parameter	**Value**	**Notes**
θ	0.0130	Chatterjee and Eyigungor (2012)
$\underline{\delta}$	0.990	Fix implied $r_f = 0.33\%$ when $\pi_S = 0$
δ	0.999	Fix B-A Spread = 2.5 b.p. when $\pi_S = 0$
$\bar{\delta}$	1.001	Fix volumes=37% when $\pi_S = 0$
λ	0.0122	Weighted Average Maturity of Debt
κ	0.0041	Average Coupon of Debt

The default cost function is set, following Chatterjee and Eyigungor (2012), to be:

$$\phi(y) = max\{d_0 y + d_1 y^2, 0\} \tag{18}$$

The signal process $\hat{y}'$ is set to be $\hat{y}' = y' + \epsilon$ with $\epsilon \sim N(0, \sigma_\epsilon^2)$, and the cost function associated with trying to observe the signal is set to $f(\pi) = f * \pi$. This leaves seven free parameters:

1. The government's inverse elasticity of intertemporal substitution (inverse EIS) ψ;
2. The government's coefficient of relative risk aversion γ;
3. The government's discount factor: β;
4. The linear default cost: d_0;
5. The quadratic default cost: d_0;
6. The conditional variance of the signal: σ_ϵ^2;
7. The cost of observing the signal: f.

These parameters are set to minimize the distance between seven empirical moments and their model counterparts. Three of them are standard moments in the sovereign default literature (see Chatterjee and Eyigungor (2012) or Bocola and Dovis (2019), for example): the mean spread, the volatility of spreads, and the mean of external debt to GDP (adjusted by the estimated face value haircut rate applied in Greece's 2012 default, since there is no recovery after default in the model).

To calculate Spanish interest rate spreads, I use data on monthly interest rates on Spanish and German government bonds from the IMF's International Financial Statistics. The spread is set to be the value of the Spanish interest rate less the value of the German one. The data on external debt comes from using the Quarterly Public Debt Statistics series of Total External Debt At Nominal Value, Total Domestic Debt At Nominal Value, and Total Debt Securities at Nominal Value with the Spanish Consolidated Financial Accounts (in which non-debt security liabilities are recorded at nominal value) produced by the Bank of Spain to derive External Debt Securities At Nominal Value. All of the above debt series correspond to the entries for the General Government sector.

I augment these three standard moments with a pair characterizing cyclical patterns in debt stocks and flows. The addition of these moments serves primarily to identify the inverse EIS ψ and the government relative risk aversion coefficient γ. Specifically, these parameters are informed by the correlation between log GDP and the debt to GDP ratio and the correlation between log GDP and the the trade balance.

The final two moments, used primarily to identify the accuracy of information and the cost of acquiring it, are the long run average bid-ask spread and the correlation of bid-ask spreads and interest rates. The monthly bid-ask spreads are calculated as described in the data section. The security-specific bid and ask price data were acquired from Bloomberg, and the weights used in aggregating across bond issues are based on security-specific data on quantity outstanding produced by the Bank of Spain.

The model is solved in Julia by value function iteration using 201 points evenly distributed in log space across six ergodic standard deviations of $ln(y_t)$ centered at 0, and 1201 points for the asset grid, equally spaced between 0 and 4. The full set of calibrated parameters is detailed in Table 3:

Table 3: Calibrated Parameters

Parameter	**Value**	**Notes**
ψ	11.73	Govt Inverse EIS
γ	4.83	Govt Relative Risk Aversion
β	0.992	Govt Discount Factor
d_0	-0.110	Linear Default Cost
d_1	0.142	Quadratic Default Cost
f	0.000125	Cost of Information (Linear)
σ_ϵ	0.037	SD of Noise in $\hat{y}$

In order to produce the countercyclical debt to GDP and a countercyclical trade balance typical of a developed economy, the inverse EIS and risk aversion coefficient must be set quite far from the standard value of 2 used in the literature. In other work focused on developed economies, such as Bocola and Dovis (2019) and Bocola et al. (2019), non-homotheticities in preferences (specifically, a subsistence level of consumption) have been used in order to try to replicate these features. In this paper, I show that adjusting the inverse EIS (and possibly the CRRA coefficient) away from 2 can be used to obtain the same result, while preserving the homotheticity of utility. The full set of targeted moments is detailed below in Table 4:

Table 4: Targeted Moments (Annualized Values)

Moment	**Period**	**Data**	**Model**
$E[B'/Y]$	Jan 1 2001 - June 30 2012	11.9%	13.5%
$\rho(B'/Y, ln(Y))$	Jan 1 2001 - June 30 2012	-0.76	-0.49
$\rho(NX/Y, ln(Y))$	Jan 1 2001 - June 30 2012	-0.78	-0.10
$E[r - r^f]$	Jan 1 2001 - June 30 2012	0.72%	0.83%
$\sigma(r - r^f)$	Jan 1 2001 - June 30 2012	1.13%	1.05%
$E[BA]$	Jan 1 2001 - June 30 2012	5.5 b.p.	5.4 b.p.
$\rho(BA, r - r^f)$	Jan 1 2001 - June 30 2012	0.84	0.80

7 Results

In this section, I explain the model's predictions about how debt crises evolve and show that they are qualitatively similar to what occurred in Spain. After that, I validate the model's key mechanism (which connects current bid ask spreads to future realizations of output) by showing that, through the lens of the model, I can apply a filter using realized bid ask spreads in order to obtain a forecast of output that is more accurate than a standard benchmark.

7.1 Crisis Dynamics

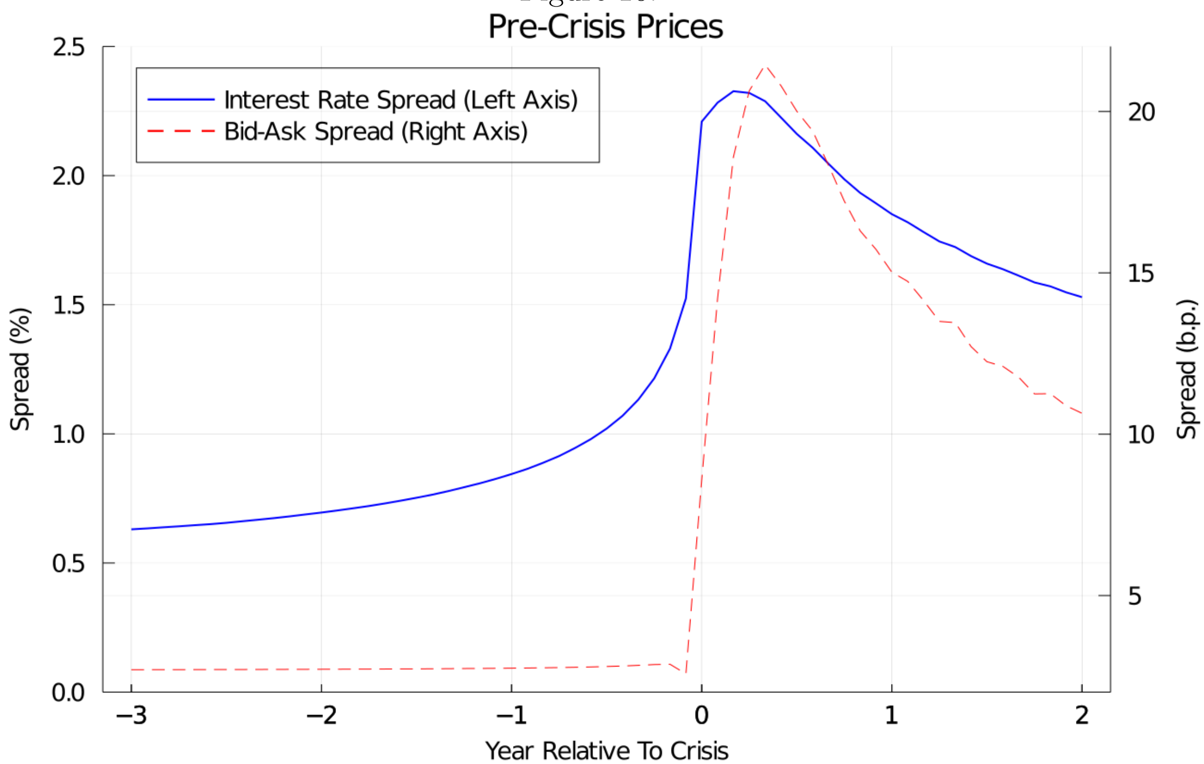
Following Bocola et al. (2019), I define a crisis in simulated data to be a period in which:

1. no crisis or default has occurred recently (within the last three years);

2. the interest rate spread rises more than one standard deviation above its mean.

The spread level implied by this rule is about 2%, which is similar to spread on Spanish government debt first achieved in January, 2011. Figure 10 plots the average paths of interest rate spreads and bid-ask spreads over the three years prior to a crisis and the two years after (conditional on not defaulting):

Figure 10:



In Figure 10, we see that interest rates rise quite sharply in the lead up to a crisis and its onset. Thereafter, that rise tapers off, and interest rates begin to fall steadily back towards their long term level. Until the actual onset of the crisis, bid-ask spreads do not react essentially at all, consistent with the patterns in the data. Recall that, in the data, bid-ask spreads were essentially unchanged from their pre-crisis levels until spreads rose above 2% at the beginning of 2011. In the simulation, once the crisis begins and interest rate spreads become significantly elevated, bid-ask spreads also rise away from zero and stay elevated throughout the next two years. Again, this is qualitatively exactly consistent with the patterns observed in the Spanish data.

Next, I discuss the losses to investors associated with these elevated bid ask spreads, measured as a change in liquidity. Specifically, I develop two related, direct measures of liquidity which I observe in the model and chart how they change over the course of a crisis. First, for any CDF F of valuations for current owners of an asset and those owners' total measure B, and any CDF G of valuations for potential buyers and their total measure A, define the maximum possible surplus from trade S^{max} and the minimum number of trades required to achieve that surplus N^{eff} as follows. First, set $v^\star$ to be smallest number satisfying:

$$B(1 - F(v^\star)) + A(1 - G(v^\star)) \leq B$$

$v^\star$ will determine the minimum valuation that any asset holder if the maximum surplus from trade is to be achieved. Then define N^{eff} as:

$$N^{eff} = \begin{cases} BF(v^\star) & \lim_{x\uparrow v^\star} F(x) = F(v^\star) \\ A(1 - G(v^\star)) & \lim_{x\uparrow v^\star} F(x) < F(v^\star) \end{cases}$$

When the CDF of valuations of initial owners is continuous at $v^\star$, every asset held by initial owners with valuations less than $v^\star$ must be traded to buyers. When the CDF of initial owners is not continuous at $v^\star$, then in addition to the assets of initial owners with valuations less than $v^\star$, some of the assets of agents with valuation exactly $v^\star$ (of which there is a point mass, in this case), must be transferred, and the total number of transfers must result in all buyers with valuation strictly greater than $v^\star$ holding an asset. Next define S^{max} as the difference between the total value of the asset under the initial allocation and the total value under the efficient allocation:

$$S^{max} = \lim_{x\downarrow v^\star} \int_x^{+\infty} vdG(v) - \lim_{x\uparrow v^\star} \int_{-\infty}^{x} vdF(v) + v^\star max\{\lim_{x\uparrow v^\star} BF(x) - A(1 - G(v^\star))), 0\}$$

The first term pertains to new holders of the asset in the efficient allocation and therefore includes every potential buyer with valuation strictly greater than $v^\star$. The second term pertains to original owners of the asset, and includes all agents with valuation strictly less than $v^\star$. When there is a point mass of potential buyers with valuation exactly equal to $v^\star$, the final term measures the value associated with assets transferred to such potential buyers.

Suppose that $S^{max}(F,B,G,A) > 0$ and $N^{eff}(F,B,G,A) > 0$. Let F', G' denote the valuation distributions of current asset holders and current non-asset holders after some given trading protocol X has been applied. Define an asset's raw liquidity under trading protocol X in circumstance F, B, G, A as:

$$L_{raw} = 1 - \frac{N^{eff}(F', B, G', A)}{N^{eff}(F, B, G, A)}$$

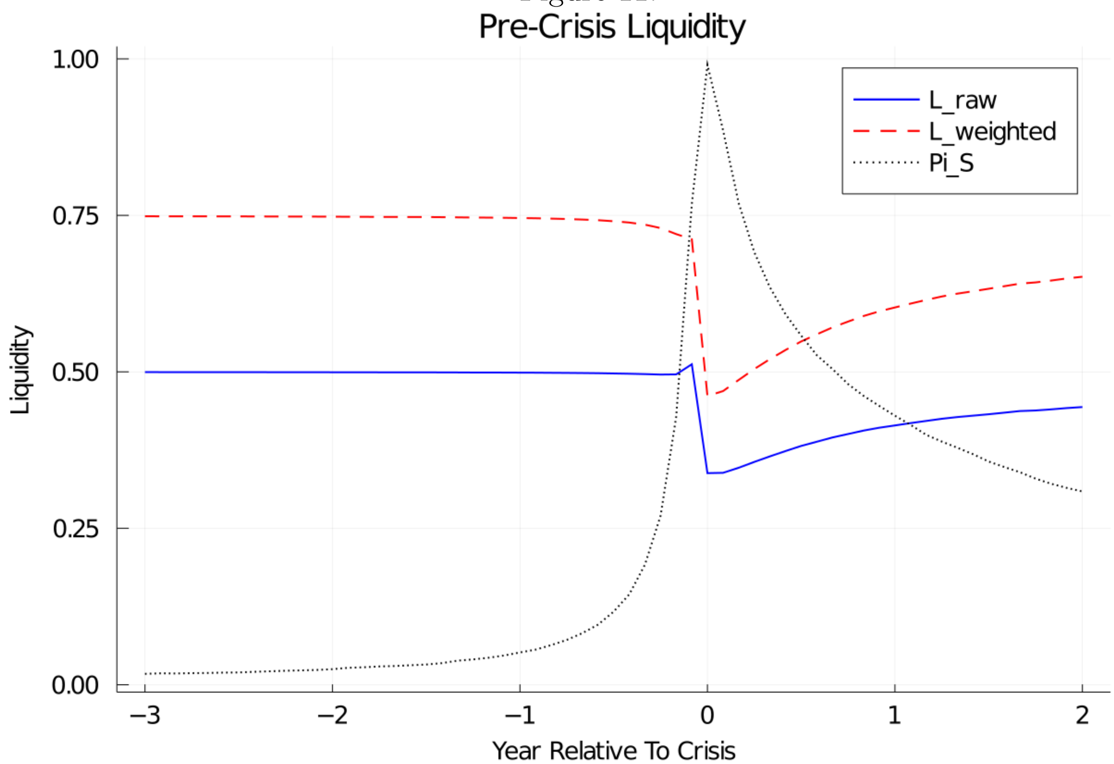
and its weighted liquidity as:

$$L_{weighted} = 1 - \frac{S^{max}(F', B, G', A)}{S^{max}(F, B, G, A)}$$

So $1 - L_{raw}$ measures the distance between the post-trade distribution under protocol X and the efficient distribution, measured in terms of the number of trades required to move from that post-trade distribution to the efficient distribution, as a percent of the initial number of trades required. Instead of measuring the raw number of trades, $1 - L_{weight}$ measures the potential surplus associated with making those remaining trades as a percent of the total maximum attainable surplus from trade. High values of L_{raw} ($L_{weighted}$) imply that the trading protocol X, in circumstance F, B, G, A, results in most trades for which there is a surplus occurring (most of the possible surpluses from trade being attained).

In other contexts, these two values might be used to measure the efficiency of different trading protocols in a single circumstance F, B, G, A or distribution of circumstances. However, for a single, given trading protocol, they also measure how easily the asset flows from those who would like to sell the asset to those who would like to buy it in various circumstances F, B, G, A. Therefore, they can be interpreted as measuring how the liquidity of an asset changes as the circumstances change. Figure 11 plots the evolution of these liquidity measures, as well as the equilibrium level of information acquisition, over the course of a crisis in the model:

Figure 11:



For reference, when there is no information acquisition, $L_{raw} = 0.5$ and $L_{weighted} \approx 0.75$. There are several important features of this picture. First, consider the period before the onset of the crisis. Over this time period, bid-ask spreads are essentially constant and equal to their level when $\pi_S = 0$. That is not because there is no information acquisition before the crisis starts. Indeed, in Figure 11, we can see that there is indeed some information acquisition over the year prior to this period. It is just that, along these paths, that information is not particularly useful and does not result in informed sellers submitting ask prices that are significantly different from those submitted by uninformed sellers. As buyers react to this information acquisition and submit relatively lower bid prices, the realized gains from trade fall, as we can see in the slight decline in $L_{weighted}$ over the sixth months leading up to a crisis. The slight rise in L_{raw} during the period prior to default is due to the signal $\hat{s}'$ in that period delivering very bad news. Since many sellers are informed at this point, many more efficient trades actually occur, although those trades to not result in large enough increase in the surplus realized to induce a similar rise in $L_{weighted}$.

After the crisis begins, information acquisition peaks and there are significant drops in both measures of liquidity. Both the number of efficient trades made in equilibrium and the surplus produced by those trades fall about a third. Furthermore, those drops in liquidity are relatively persistent, and efficiency of trade suffers for years, even as information acquisition falls away from its peak.

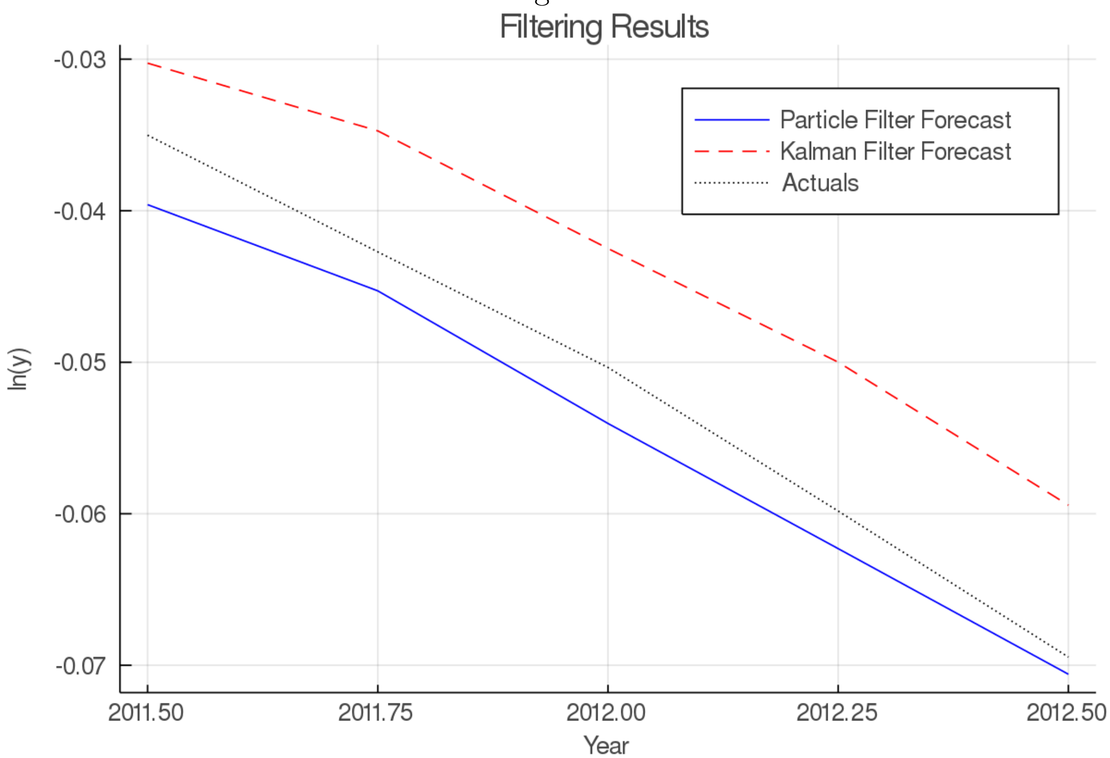
7.2 Forecasting Output

I now turn to validating the model's main mechanism, the connection between current realizations of bid-ask spreads and future realizations of output. To do this, I employ the particle filter, following Bocola (2016) and Bocola and Dovis (2019). I now describe the process of applying this filter once.

Specifically, in each quarter, based on a prior for the joint distribution of the persistent value of the output process in the first month of the quarter y and the debt level b at the end of the last month of the previous quarter, I randomly generate a large number of sequences of persistent monthly output y_t and transitory monthly output m_t so that the values of output associated with the current quarter sum to the actual observed value of quarterly output in that quarter (so six values total per quarter, but only five degrees of freedom). I then simulate those three months of decisions by the government to obtain three months of interest rate spreads, bid-ask spreads, and default decisions. I then use these outcomes to calculate the likelihood of each path. Here, I place a large enough weight on the simulated default decision matching the data to ensure that paths leading to default have 0 likelihood. In calculating the likelihood contributions of the simulated interest rate spreads and bid-ask spreads, I assume that there are small (i.e. have variance equal to $1-2\%$ of the corresponding data series's variance) i.i.d., normally distributed measurement errors. Using these likelihood values for the current set of paths, I produce a posterior distribution on y, b where y is the value of the persistent component of output in the first month of the next quarter. This posterior distribution is used to resample paths at the beginning of the filtering process in the next quarter.

Given only the up to date distributions of values of y at the end of each quarter produced by this filter, I then produce a forecast of the next quarter's output. I compare this to a benchmark forecast, which is the value predicted by the Kalman Filter using all information available at that time. The results of this process, using 2011Q1 as the initial quarter (and the data value of debt to GDP in that period to set the initial b), are plotted below in Figure 12:

Figure 12:



As can be seen in Figure 12, realized bid ask spreads and interest rate spreads, filtered through the lens of the model, provide additional information on the state of the economy. In order to match the realized bid-ask spreads, the model requires that news about tomorrow be consistently relatively poor. However, it does not require that news to be consistently too poor (i.e. so that the Particle Filter forecast misses by more below than the Kalman Filter forecast does above).

8 Conclusion

In this paper, I have documented some novel facts about the relationships between bid-ask spreads, realized trading volumes, and interest rate spreads in Spain. I have shown that realized trading volumes are uniformly low when bid-ask spreads are elevated, and that the relationship between bid-ask spreads and interest rate spreads differs by the level of interest rate spreads. Specifically, bid-ask spreads are not affected by variation in interest rate spreads when default risk is low. However, they react quite sharply once default risk rises to significant levels.

In order to rationalize these relationships, I built a model of sovereign default with frictional secondary markets where bid-ask spreads are produced by the presence of traders with private information. This private information pertains to a future realization of output (which is valuable to traders because the government's default and borrowing decisions depend on the realization of output). After characterizing the nature of equilibria in secondary markets, I calibrate the model to the experience of Spain during the Eurozone Debt Crises. I then use the model to illustrate the dynamics of crises and measure how liquidity changes over the course of the average crisis. Finally, I validate the model's mechanism by showing that incorporating information about bid-ask spreads into forecasts of future output by using the model-implied relationship between realized secondary market behavior and actual future output improves the accuracy of those forecasts.

While the trading protocol used in this paper was sufficient to replicate the patterns predicted by the presence of asymmetric information, it does not directly incorporate a notion of financial intermediaries, which are known to have played a significant role in secondary markets for government debt in Europe. The role of their activities and interactions are exactly the focus of Chaumont (2018). The implications of combining the two mechanisms are unknown, and present an promising avenue for future research.

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