Sovereign Default and Government Reputation

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Abstract

In this paper, I build a flexible theoretical model of sovereign borrowing, default, and renegotiation with borrower reputation. There is asymmetric information about the government's "type", and reputation is the market belief that it is "responsible" and therefore less likely to default. I calibrate the model using data on how countries' credit histories affect the prices they face and validate its predictions about the effects of borrowing on interest rate spreads in the data. Using the model, I show that countries that have recently defaulted have poor reputations because they rapidly run up their debts prior to default, not because the default decision itself is revealing. I also show that, for countries facing non-trivial levels of default risk, the reputational benefits of repayment are less than 0.5 basis points of consumption. Policies that disrupt the signalling motives induced by asymmetric information, such as transparency initiatives or fiscal rules, can have substantial negative implications for welfare (losses of $0.23\% - 0.85\%$ of permanent consumption), because they lead to increased overborrowing by the government.

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1 Introduction

One key determinant of interest rates on government debt is whether the issuing country has a good reputation for repaying its debts. There are countries like Germany, which have built strong reputations for being frugal and consistently paying back their debts, and there are countries like Argentina, which have become notorious for how frequently they default. Reinhart et al. (2003) show that such patterns are both systematic and economically meaningful. Having been in default more frequently and more recently is correlated with markets' perceived likelihood a country will default again. However, if countries avoid defaulting for long enough, they appear to "graduate" and shed their old reputation (Qian et al., 2015).

There is evidence in statements going back hundreds of years that policy makers are interested in building and maintaining a good reputation for their countries in financial markets. For example, right after the United States Constitution was ratified in 1789, Alexander Hamilton pushed for the new Federal Government to assume the debts of the states that had been accrued in waging the American Revolution. His reasoning was that, by shouldering a large debt burden and steadily, consistently paying it back, the country could establish a good reputation in international markets, ensuring that it could borrow at cheaper rates should it need to, in the future (Hall et al., 2021).

More recently, Cruces and Trebesch (2013) and Catão and Mano (2017), among others, have found that governments of Emerging Market Economies that have recently restructured debts tend to face significantly elevated interest rates. They show that these effects are robust to controlling for macroeconomic and political factors. In this paper, I construe this as evidence of reputational effects and ask, "What story or mix of stories is producing them?" Does default ruin a country's reputation? Or does having a terrible reputation lead to default because it makes rolling over debt very expensive? Alternatively, do countries wreck their reputation simply by accumulating enough debt that default becomes remotely likely? All three of these would produce the same associations we see in the data.

The purpose of this paper is to build a theory of how governments build and maintain a good reputation, how they lose it, and how concerns about this reputation influence their borrowing, default, and renegotiation decisions. In doing so, I fill a gap in the literature on sovereign default: the role of reputation. Quantitative research on this issue has been sparse, but that does not reflect doubts that reputation is important. Rather, it follows from a limitation of existing models to capture reputational effects in a tractable way and discipline them using data.

In this paper, I build a state of the art model of sovereign borrowing, default, and renegotiation which incorporates a notion of reputation. The model contains a number of features that the literature has shown to be important for matching the data on sovereign borrowing, including long term debt and a flexible renegotiation bargaining protocol.

In my model, reputation is the market belief about whether a government is believed to be "responsible" or "irresponsible". The true "type" of the government is known only to the government itself. In this model, the government makes a wide variety of decisions, including how much to borrow, whether to default, and what type of renegotiation offers to make to lenders (or accept from them). All of those decisions can affect its reputation. This flexibility about which decisions affect reputation is a key theoretical improvement over the existing literature (which has focused almost exclusively on the default vs. repayment margin).

The differences between the preferences of the two types will determine the differences in how they behave and therefore how historical choices can be informative about future outcomes. I show how the post-restructuring patterns estimated by Cruces and Trebesch (2013) identify the parameters controlling the stochastic government type process and the flow of information in the model. In order to discipline my model, I require that it replicate those same patterns, which describe how historical restructurings affect current spreads up to seven years after the fact, as well as match a relatively standard set of targeted moments.

One key prediction of the model is that a country's reputation is negatively correlated with its interest rate. In order to validate the model, I test this prediction. Since reputation cannot be directly observed in the data, I exploit another prediction of the model, that borrowing decisions affect a country's reputation. My calibration strategy left the reputational consequences of borrowing decisions on interest rates as an untargeted moment (since it directly targeted only the correlations between restructuring histories and interest rates induced by the inclusion of types and reputation). To validate the model, I first use it to measure how issuance choices affect reputation. Then I use data on debt issuances for a large set of countries to construct a measure of those countries' reputations. This model-filtered reputation provides significant additional explanatory power in reduced form estimates of the determinants of interest rate spreads and near term default probabilities. I consider this evidence that the out of sample predictions made by my model are quantitatively sound.

After calibrating the model and validating some of its key out of sample predictions, I describe its key features and the effects of information asymmetry on government behavior. One key result that emerges is that the default decision in and of itself is not very informative along the equilibrium path. It is the case, in the model, that countries which have recently defaulted have very poor reputations, but this is because they had run their reputation into the ground by borrowing very large amounts prior to defaulting. This rapid borrowing both destroys the country's reputation and raises its probability of default, so the vast majority of countries which do default already have poor reputations.

The signalling motives induced by asymmetric information about the government's type are a key driver of this result. The responsible type faces powerful incentives to prove to lenders that it can indeed be trusted, and it responds by borrowing substantially less than it would if its type were public information. Of course, the irresponsible type does sometimes imitate the responsible type in order to take advantage of the higher prices lenders offer the responsible type, especially when debt levels are low. However, as the country accumulates more and more debt, this imitation becomes too costly for the irresponsible type. At that tipping point, it gives up and rapidly accumulates debt before eventually defaulting.

In a related result, I quantify the potential reputational benefits of repayment for countries facing non-trivial levels of short term default risk. I find that countries exposed to even 1% default risk have poor reputations, and the average value of the reputation gained by repayment is less than 0.005% of consumption. For countries facing higher levels of default risk, these gains are even smaller. Furthermore, this is not because the model implies having a good reputation is never valuable. Indeed, it is possible for the value of a good reputation to be worth almost 2% of consumption. However, countries whose reputations are worth a lot almost invariably have low debt and are far from default. That reputation is valuable to them precisely because it lets them borrow at favorable prices, and they have significant room to borrow. By the time the possibility of default is truly on the horizon, countries have very little fiscal space, so the value of being able to borrow at higher prices is negligible. This suggests that, in policy debates about the benefits of helping a country avoid default, reputational concerns should not play the outsize role they often have.

While the motive for signalling via the default vs. repayment margin is muted, the signalling motives affecting borrowing decisions are powerful. In the long run, they lead to the responsible type borrowing about three quarters as much as it would were its type public knowledge (and defaulting less than one fifth as often). While they also impact the choices of the irresponsible type, the effects are smaller. The overall strength of these motives has important implications for the welfare effects of policies that disrupt them, such as transparency initiatives and fiscal rules. I explore these in the next section of the paper.

In this section of the paper, I measure the welfare losses associated with weakening the signalling motives induced by asymmetric information. Completely removing signalling motives by making government type publicly observable (this is one explicit interpretation of what a transparency initiative does) leads to losses of 0.23% of permanent consumption for the irresponsible type, 0.50% for the responsible type, and 0.85% for a representative consumer. This result follows from the presence of overborrowing in the setting with transparency and the strength of signalling incentives in the baseline model. Long term debt introduces dilution motives that make the equilibrium allocation without asymmetric information inefficient (see Aguiar and Amador (2019) for a detailed discussion of this phenomenon). This inefficiency manifests as overborrowing by the government. The signalling motives under asymmetric information cause the government to borrow much less, partially correcting the underlying inefficiency. As a result, default, which is quite costly for both government payoffs and consumer welfare, occurs significantly less often. While the government does suffer from the inability to run up its debt quite as fast, relative to the case with transparency, it issues debt at prices which are uniformly higher than they would be in that case. This reduces the losses it suffers from only reaching lower overall borrowing levels.

This result also has implications for how policymakers evaluate fiscal rules. Insofar as a fiscal rule that limits debt accumulation is ever actually binding, it will generally constrain the irresponsible type's choices more frequently than the responsible type's, since the irresponsible type in general prefers to borrow more. By distorting the irresponsible type's behavior and forcing it to borrow less, the rule can make it very expensive for the responsible type to prove to lenders that it is indeed the responsible type, because it must pull back even more on borrowing. If signalling its type becomes too expensive, the responsible type may give up and end up borrowing more and defaulting more often.

2 Literature Review

This paper builds on the literature studying the effects of default on a government's reputation. Closely related papers here include Cole et al. (1995), English (1996), Egorov and Fabinger (2016), Alfaro et al. (2005), D'Erasmo (2011), Amador and Phelan (2021), Amador and Phelan (2023), and Morelli and Moretti (2023). Each considers a sovereign default model based on the classic paper of Eaton and Gersovitz (1981), extended so that the government has private information about its type (which determines its preferences and/or choice set). Almost all of these papers only consider how the default decision reveals information about the country's type. In contrast, I allow all the government's actions (borrowing, default, renegotiation, and restructuring) to be informative to lenders. I then use the data to discipline the parameters governing how informative each of the government's actions is. I show that this flexibility matters. Indeed, once the model is calibrated to the data, the default decision itself is actually not very informative (in equilibrium). Instead, the borrowing decisions prior to the default convey the vast majority of the information. One notable exception to the literature's focus on the default decision is Morelli and Moretti (2023), who allow reports (or misreports) of inflation, as well as default, to affect the government's reputation.[1]

This paper also draws on work about consumer default. Indeed, Chatterjee et al. (2020) is the closest paper, methodologically, to mine. They have a similar focus on reputation about

[1] The paper studies why misreports of Argentina's inflation affect spreads on its dollar-denominated debt. To focus on this margin, they assume fiscal policy is set by an agent, e.g. congress, with the same information set as lenders (i.e. does not observe the government's type), so borrowing decisions provide no information.

the preferences of the borrower, with a goal of explaining the basis for credit scores and how they affect behavior. A key methodological innovation of this paper is introducing preference shocks to ensure every feasible action is played with nonzero probability in equilibrium. Therefore, there is no such thing as feasible actions off the equilibrium path, and the evolution of beliefs is fully determined by actual equilibrium strategies. Thus there is no need to arbitrarily specify how beliefs evolve off of the equilibrium path, (such as in Egorov and Fabinger (2016) or D'Erasmo (2011) for example). In this paper, I rely on the same technique. In addition, I add long term debt and endogenous renegotiation, as well as enriching the type space to allow for the cost of default as well as the patience rate to vary by type. I show these additions are important for the model's performance in the sovereign debt setting.

This paper also builds on the overall quantitative sovereign default literature as well as the subset focused on endogenous renegotiation. This literature is based on the classic setting of Eaton and Gersovitz (1981). Key early papers include Aguiar and Gopinath (2006), Arellano (2008), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012). One key insight of this work is that incorporating long term debt is critical for being able to match the levels of debt and levels and volatility of interest rate spreads observed in Emerging Market Economies. These models with long term debt provide a workhorse model that many branches of the literature build on.[2] I too build on this baseline environment by introducing asymmetric information and explicitly modelling the renegotiation process.

The renegotiation process has also been a focus of the quantitative literature. One early contribution here is Yue (2010), which uses a Nash Bargaining framework to study how the determination of recovery rates affects the government's borrowing and default behavior. My paper builds on two newer, related papers that study renegotiation, Benjamin and Wright (2013) and Dvorkin et al. (2021), both of which study why there exist delays between default and the completion of the renegotiation process. To do this, they model the renegotiation process as a game with an alternating offers structure, building on the general bargaining en-

[2]Among these are papers focused on the government's choice of debt maturity (Arellano and Ramanarayanan (2012), Sánchez et al. (2018), Bocola and Dovis (2019), and Dvorkin et al. (2021)), papers focused on the interaction between borrowing and default decisions and a domestic production economy (Mendoza and Yue (2012), Bocola (2016), and Gordon and Guerron-Quintana (2018)), and papers focused on the role of rollover risk and self-fulfilling crises (Conesa and Kehoe (2017) and Bocola and Dovis (2019)).

vironment described by Merlo and Wilson (1995). They focus on explaining the existence of delays as well as the role of maturity extensions in enabling more efficient settlements. While this paper largely abstracts from the causes of delays and focus on the patterns observed after the renegotiation process is completed, I incorporate this bargaining protocol into my model because it is particularly well suited to a setting with asymmetric information.

Finally, there are many empirical studies of how past defaults and restructurings affect interest rate spreads, including Dell'Ariccia et al. (2006), Borensztein and Panizza (2009), Cruces and Trebesch (2013), and Catão and Mano (2017). Overall, these find significant, positive effects on spreads in the first two years after a default. Cruces and Trebesch (2013) also study the relationship between post-restructuring spreads and the scale of debt relief, "haircuts". Whereas most prior papers found effects lasting less than three years, they find that restructuring outcomes can have significant effects for up to seven years. While the effects of restructurings with small or average haircuts may attenuate to zero within the first couple years, ones with higher haircuts can have economically significant effects for up to seven years. Catão and Mano (2017) find similar results using lower frequency data. These papers focus primarily on describing the patterns in spreads after restructurings observed in the data and are agnostic as to why they arise. I take the existence of these patterns as given, and build a structural model whose parameters can be disciplined using them.

3 Model

Time is discrete and infinite. There is a small open economy populated by a representative consumer and a government policymaker. Both have expected utility preferences and period flow utility given by $u(c)$ with u a nice function. The public, exogenous state of the world is s, which is a Markov Process (and governs the country's endowment $y(s)$). The private, exogenous state of the world includes T, the government policymaker's type. This type is independent of s and also a Markov Process. T is persistent (but not permanent) and known only to the government policymaker, but its transition rule is public knowledge.

The government policymaker's type T determines their current discount factor β_T and cost of default in the current period $\phi_T(s, d_t)$. For simplicity, I assume that type maps to a single, specific pair of discount factor β_T and default cost $\phi_T(s, d_t)$. However, it is possible to relax this assumption of perfect correlation and allow type to instead define a probability distribution over such pairs. The government policymaker understands that their type may change over time, and they incorporate that possibility into its optimization process. Type encodes characteristics and/or private knowledge of the government policymaker. Again for simplicity, I assume that whenever there is a change in the public identity of the policymaker (for example, when an election results in a new governing party or governing coalition), the underlying type of the new policymaker matches the underlying type of the old policymaker. It is straightforward to extend this setting to incorporate the possibility of observable "changes in head of government, governing party, or governing coalition," at which time the transition rule for the policymaker's type differs from the rule in normal times (i.e. when there is no such change). Furthermore, I assume that lenders receive no information about the policymaker's type except information conveyed by the policymaker's decisions in the model. Again, it is straightforward to extend this setting to allow for some exogenous flow of information, independent of the policymaker's decisions, via a publicly observable signal. From now on, I will refer to this government policymaker simply as "the government."

The private, exogenous state of world also includes preference shocks ϵ, η, or ν for the government that are independent of the its type and i.i.d. over time. These preference shocks enter additively in the government's decision problems. They are unbounded and therefore ensure that every feasible action is played with nonzero probability in equilibrium. Since the equilibrium concept will be Bayesian Perfection, this is valuable because it ensures that I never need to arbitrarily specify how beliefs evolve off the equilibrium path. Let π denote lender beliefs about T, which are updated using Bayes Law and the transition rule for T. Whenever an action a is observed, it triggers a belief update Γ^A:

$$\Gamma^A(a, \pi | s, b)(T_0) = \frac{\pi(T_0) Pr(a^\star(s, T_0, \epsilon, \pi, b) = a)}{\int_{\mathscr{T}} \pi(x) Pr(a^\star(s, x, \epsilon, \pi, b) = a) dx}$$

At the end of a period, beliefs are updated to account for possible changes in the government's type between periods. This second kind of update will be denoted as $\Gamma(\pi)$.

The government may borrow from a continuum of international lenders using a defaultable long term bond. There is a finite set $\mathscr{B}$ of values that the government's debt level b can take. Following Chatterjee and Eyigungor (2012) and Hatchondo and Martinez (2009), I model this debt as a contract promising a stream of exponentially declining coupon payments. Specifically, at time t, a unit of the bond promises to pay $(1-\lambda)^{t+l-1}(\lambda+\kappa)$ of the consumption good in period $t+l$. If the government chooses to default, the country enters financial autarky and begins suffering a flow utility penalty of $\phi_T(s, d_t)$. This penalty depends on the exogenous, public state of the world s, the government's type T, and whether the country entered default in the current period d_t (as opposed to having defaulted in some previous period but not yet completed the renegotiation process). In order to resolve a default, the government must negotiate an agreement with bondholders.

There is an issuance cost $i(s, b, \pi', b')$ (possibly 0) incurred when the government adjusts its debt level. This is standard in models with long term debt and positive recovery rates (see Dvorkin et al. (2021) or Chatterjee and Eyigungor (2015) for example). If it is omitted and recovery rate on debt is positive, the government has an incentive to issue enormous amounts of debt the period before a default in order to fully extract the value of legacy creditors' holdings. Since this type of "maximum" dilution behavior is counterfactual, issuance costs are added to the model to prevent it from occurring in equilibrium. Quantitatively, the amount of resources spent financing the issuance costs ends up being small.

3.1 Repayment

If the country enters the period in good standing, the timing of events is as follows:

1. The states s, T, and $\epsilon = \{\epsilon^D, \{\epsilon^R(b')\}_{b' \in \mathscr{B}}\}$ are realized.
2. The government chooses whether to default (d). This triggers belief update $\Gamma^D(d, \pi|s, b)$.
    - If the government has chosen not to default, it chooses the new level of debt b'. A belief update based on that decision $\Gamma^R(b', \pi|s, b)$ occurs.
    - If the government has chosen to default, it enters bad standing and possibly begins negotiations with lenders. The exact sequence of events and actions which occur in

this case will be specified later, in the sections on renegotiation and restructuring.

3. A belief update based on the transition rule for T, $\Gamma(\pi)$, occurs (updating π to be the prior belief at the beginning of the next period).

At the beginning of the period, if the country is not in default, its problem is:

$$V(s,T,\epsilon,\pi,b) = \max_{d\in\{0,1\}} (1-d)V^R(s,T,\epsilon,\hat{\pi},b) + d(V_0^D(s,T,\hat{\pi},b)+\epsilon^D)$$

$$\text{where:}$$

$$\hat{\pi} = \Gamma^D(d,\pi|s,b)$$

where V^R is the value of repayment, Γ^D describes how lender update their beliefs based on the default decision, and V_0^D is the value of entering default. Here, the government decides whether to default, understanding the reputational consequences of that choice. If it chooses to repay its debt, it solves:

$$V^R(s,T,\epsilon,\pi,b) = \max_{c,b'\in\mathscr{B}} (1-\beta_T)u(c) + \beta_T\mathbb{E}[V(s',T',\epsilon',\pi',b')|s,T] + \epsilon^R(b')$$

$$\text{such that}$$

$$c + (\lambda+\kappa)b = y(s) + q(s,\pi',b')(b'-(1-\lambda)b) - i(s,b,\pi',b')$$

$$\text{where:}$$

$$\pi' = \Gamma\Big(\Gamma^R(b',\pi|s,b)\Big)$$

$\Gamma^R(.)$ represents the law for updating lender beliefs conditional upon witnessing specific borrowing decisions. In this problem, the government chooses its optimal borrowing level b' taking into account how that choice affects both the revenue raised in the auction today $q(.)(b'-(1-\lambda)b)-i(.)$ and the continuation value that it will receive in the future $\mathbb{E}[V(.)|s,T]$. In a model without reputation, the choice of b' would affect continuation values by changing future constraint sets, and the price by changing next period's policies. My model embeds those channels as well as an additional one, reputation. Different choices of debt induce different belief updates by lenders, which have effects on current prices and continuation values that are independent of the actual physical debt level.

Before moving onto discuss the renegotiation process, I detail the belief updates and prices while in good credit standing. Since lenders are Bayesian, Γ^D and Γ^R are given by:

$$\Gamma^D(d, \pi|s, b)(T_0) = \frac{\pi(T_0)Pr(d^\star(s, T_0, \epsilon, \pi, b) = d)}{\int_{\mathscr{T}} \pi(x)Pr(d^\star(s, x, \epsilon, \pi, b) = d)dx}$$
$$\Gamma^R(b', \pi|s, b)(T_0) = \frac{\pi(T_0)Pr(b'^\star(s, T_0, \epsilon, \pi, b) = b')}{\int_{\mathscr{T}} \pi(x)Pr(b'^\star(s, x, \epsilon, \pi, b) = b')dx}$$

Since lenders are competitive and risk neutral, the price of a bond will be exactly equal to the expected present value of the sequence of payments holding the bond entitles them to. The value of this sequence of payments depends on the exogenous state of the world s, lenders' prior belief about the government's type at the beginning of the next period π', and the country's debt level b'. Repayment prices $q(s, \pi', b')$ are defined recursively by:

$$q(s, \pi', b') = \frac{1}{R}\mathbb{E}\left[\int_{\mathscr{T}} \Big(d'q^D(s', \pi''_D, b') + (1-d'))(\lambda + \kappa + (1-\lambda)q(s', \pi''_R, b''))\Big)\pi'(T')dT'|s\right]$$

where:

$$d' = d^\star(s', T', \epsilon', \pi', b')$$
$$\pi''_D = \Gamma^D(1, \pi'|s', b')$$
$$b'' = b'^\star(s', T', \epsilon', \pi', b')$$
$$\pi''_R = \Gamma\Big(\Gamma^R\big(b'', \Gamma^D(0, \pi'|s', b')|s', b'\big)\Big)$$

This definition illustrates how reputation is reflected in prices. First, it changes lenders' perceptions of each type being the decision maker tomorrow, reflected in the integration with respect to $\pi(T')dT'$. Second, given the government's type, its optimal policies in the following period d' and b'' depend on the reputation it has at the beginning of the period, so the terms being integrated are also directly affected by reputation.

3.2 Renegotiation

I follow Dvorkin et al. (2021) in using an alternating offers structure of renegotiation. If the government enters a period in bad standing, the first events of that period are the realization of s and T. After that, (or if the government had entered the period in good standing but then defaulted), the following occur:

1. With constant probability ψ, an opportunity for renegotiation arises this period.
2. If an opportunity for renegotiation arises, the identity of the party proposing the deal $P \in \{G, L\}$ is drawn with μ_G the probability that the proposer is the government.

If no opportunity for renegotiation arises, then the period ends and lenders update their beliefs to account for possible type switches between periods (using $\Gamma(\pi)$). If an opportunity arises, the renegotiation process begins. The proposer makes a take it or leave it offer to the other party. Offers consist of an ex post unit value to lenders Q, so a lender holding a unit of the bond will receive value Q if the deal is agreed. There is a finite set $\mathscr{Q}$ of values that Q can take. If the other party accepts the deal Q, the country enters the restructuring process committed to deliver a total value of Qb (where b was the level of defaulted debt) to lenders this period. If the deal is rejected, the country remains in default and the period ends.

There are preference shocks η_D^P associated with choices of the proposer and preference shocks η_D^R associated with choices of the party receiving the offer. The government's are private and serve the same purpose that they did in the repayment problem (ensuring that all feasible actions are played with nonzero probability). The lender's are public and are included to ensure computational tractability. The specific timing of these events is:

1. The proposer's preference shocks $\eta_D^P = \{\eta^O(Q)\}_{Q \in \mathscr{Q}}$ are realized.
2. The proposer chooses which offer Q to make. If the government is proposing the deal, a belief update $\Gamma_G^Q(Q, \pi | s, b)$, based on this decision, occurs.
3. The receiver's preference shocks $\eta_D^R = \{\eta^Y, \eta^N\}$ are realized.
4. The receiving party chooses A, whether to accept (Y) or reject (N) the offer. If the lender is the proposer, a belief update $\Gamma_L^Q(A, \pi | s, b, Q)$, based on this decision, occurs.
5. If offer was accepted, the restructuring process begins. If the offer was rejected, a belief update $\Gamma(\pi)$ based on transition rule for T occurs.

Before describing exactly how the debt is restructured once a deal has been reached, I detail the renegotiation process itself more fully. The value of a government in default is:

$$V^D(s,T,\pi,b) = \psi(\mu_G \mathbb{E}[V_G^D(s,T,\eta_D^P,\pi,b)] + (1-\mu_G)V_L^D(s,T,\pi,b)) + (1-\psi)V_N^D(s,T,\pi,b)$$

where $V_P^D(.)$ is the government's value in default when an opportunity to renegotiate arises and party P is the proposer, and $V_N^D(.)$ is the value to the government in default no opportunity arises. The last is simply:

$$V_N^D(s,T,\pi,b) = (1-\beta_T)(u(y(s)) - \phi_T(s,0)) + \beta_T \mathbb{E}[V^D(s',T',\pi',b)|s,T]$$
$$\text{where:}$$
$$\pi' = \Gamma(\pi)$$

Similarly, the value to the lender of holding a bond at the beginning of a period is:

$$q^D(s,\pi,b) = \psi(\mu_G \bar{q}_G^D(s,\pi,b) + (1-\mu_G)\mathbb{E}[\hat{q}_L^D(s,\eta_D^P,\pi,b)]) + (1-\psi)q_N^D(s,\pi,b)$$

where $\bar{q}_G^D(.)$ is the lenders' value if an opportunity to renegotiate arises and the government is the proposer, $\hat{q}_L^D(.)$ is their value if an opportunity arises and they are the proposer, and $q_N^D(s,\pi,b)$ is their value if no opportunity arises. The last is simply:

$$q_N^D(s,\pi,b) = \frac{1}{R}\mathbb{E}[q^D(s',\pi',b)|s]$$
$$\text{where:}$$
$$\pi' = \Gamma(\pi)$$

When an opportunity for renegotiation arises and the government is the proposer, it solves:

$$\begin{aligned} V_G^D(s,T,\eta_D^P,\pi,b) = \max_{Q \in \mathcal{Q}} & Pr(A_L = 1|s,\hat{\pi},b,Q)\mathbb{E}[V^{RS}(s,T,\nu^{RS},\hat{\pi},Qb)] \\ & + Pr(A_L = 0|s,\hat{\pi},b,Q)V_N^D(s,T,\hat{\pi},b) + \eta^O(Q) \\ & \text{where:} \\ \hat{\pi} = {} & \Gamma_G^Q(Q,\pi|s,b) \end{aligned}$$

When the government makes an offer, lenders beliefs are updated from π to $\hat{\pi}$ using the belief update function $\Gamma_G^Q(.)$. The first term in the maximization, $V^{RS}(.)$, represents the value to the government of entering the restructuring process with this updated value of reputation and obligated to deliver a total payment of Qb to lenders, weighted by the probability that lenders accept the offer Q, $Pr(A_L = 1|.)$. The second term in the maximization, $V_N^D(.)$, represents the value to the government of remaining in default with that same updated value of reputation weighted by the probability that lenders reject the deal Q, $Pr(A_L = 0|.)$. The final term is just the preference shock for choosing to propose the offer Q.

After the government makes an offer and lenders update their beliefs based on that decision, the receiver's vector of preference shocks $\eta_D^R = \{\eta^Y, \eta^N\}$ is drawn and lenders solve:

$$\hat{q}_G^D(s, \eta_D^R, \pi, b, Q) = \max_{A_L \in \{0,1\}} A_L\Big[Q + \eta^Y\Big] + (1 - A_L)\Big[q_N^D(s, \pi, b) + \eta^N\Big]$$

If the deal is agreed, the value to the lender will simply be $Q + \eta^Y$. If it is not agreed, then the lender retains their claim and the value associated with it, $q_N^D(s, \pi, b)$, and receives the preference shock associated with rejecting the deal η^N. The ex ante value to lenders when the government is chosen to propose a deal is then:

$$\begin{aligned} \bar{q}_G^D(s, \pi, b) &= \mathbb{E}[\hat{q}_G^D(s, \eta_D^R, \hat{\pi}, b, Q_G^\star(s, T, \eta_D^P, \pi, b))] \\ &\text{where:} \\ \hat{\pi} &= \Gamma_G^Q(Q_G^\star(s, T, \eta_D^P, \pi, b), \pi | s, b) \end{aligned}$$

If lenders, on the other hand, are the party chosen to propose a deal, then they solve:

$$\begin{aligned} \hat{q}_L^D(s, \eta_D^P, \pi, b) = \max_{Q \in \mathcal{Q}} & \, Pr(A_G = 1 | s, \pi, b, Q)Q \\ & + (1 - Pr(A_G = 1 | s, \pi, b, Q))q_N^D(s, \hat{\pi}, b) + \eta^O(Q) \\ & \text{where:} \\ \hat{\pi} &= \Gamma_L^A(A_G, \pi | s, b, Q) \end{aligned}$$

Once lenders make an offer, the government decides whether to accept it by solving:

$$\hat{V}_L^D(s, T, \eta_D^R, \pi, b, Q) = \max_{A_G \in \{0,1\}} A_G(\mathbb{E}[V^{RS}(s, T, \nu^{RS}, \hat{\pi}, Qb)] + \eta^Y)$$
$$+ (1 - A_G)(V_N^D(s, T, \hat{\pi}, b) + \eta^N)$$

where:

$$\hat{\pi} = \Gamma_L^A(A_G, \pi | s, b, Q)$$

If the government accepts the offer ($A_G = 1$), it gets the expected value $\mathbb{E}[V^{RS}(.)]$ of restructuring its debt when promising to deliver value Qb to lenders, and the taste shock for acceptance. If it rejects the offer ($A_G = 0$), it gets the value of remaining in default, $V_N^D(.)$ and the taste shock for rejection. In both cases, lenders update their beliefs based on the government's choice. The ex ante government value when lenders are the proposer is:

$$V_L^D(s, T, \pi, b) = \mathbb{E}[\hat{V}_L^D(s, T, \eta_D^R, \pi, b, Q_L^\star(s, \eta_D^P, \pi, b))]$$

The initial value of default considered by the government in the repayment problem is:

$$V_0^D(s, T, \pi, b) = V^D(s, T, \pi, b) + \phi_T(s, 0) - \phi_T(s, 1)$$

The difference in penalties appears in this definition because the various definitions of other default value functions assume that the country defaulted in a prior period.

3.3 Restructuring

Once a deal Q is agreed, the government has committed to deliver lenders $W = Qb$ value and moves to the restructuring process (i.e. deciding how to deliver W). At this point, it regains access to international markets and can use a new auction of debt to fund W. Also, another vector of preference shocks $\nu = \{\nu^{RS}(b')\}_{b' \in \mathscr{B}}$ is realized and the government solves:

$$V^{RS}(s,T,\nu^{RS},\pi,W) = \max_{c,b' \in \mathscr{B}} (1-\beta_T)(u(c) - \phi_T(s,0)) + \beta_T \mathbb{E}[V(s',T',\epsilon',\pi',b')|s,T] + \nu^{RS}(b')$$

$$\text{such that}$$

$$c + W = y(s) + q(s,\pi',b')b'$$

$$\text{where:}$$

$$\pi' = \Gamma\Big(\Gamma^{RS}(b',\pi|s,b)\Big)$$

This renegotiation/restructuring protocol is more flexible than one restricted to face value haircuts (i.e. there is only a change of the debt level from b_{old} to b_{new}; see D'Erasmo (2011) or Sunder-Plassmann (2018) for examples of this). It allows for an exchange of old bonds for new bonds as well as a cash transfer at the time of the exchange. Since such transfers are very common elements in real world restructuring deals,[3] the correlation between restructuring outcomes and future interest rates will be key for identifying the parameters of my model, it is important to allow for them. Furthermore, it allows measured haircuts in the model to be different from the face value reductions in the debt.

3.4 Equilibrium

An stationary recursive competitive equilibrium for this environment consists of:

1. Value functions $V, V^R, V_0^D, V^D, V_G^D, V_L^D, \hat{V}_L^D, V_N^D, V^{RS}$;
2. Price functions $q, q^D, q_N^D, \bar{q}_G^D, \hat{q}_G^D, \hat{q}_L^D$;
3. Policy functions $d^\star, b'^\star, Q_G^\star, Q_L^\star, A_G^\star, A_L^\star, b'^\star_{RS}$;
4. Belief update functions $\Gamma^D, \Gamma^R, \Gamma_G^Q, \Gamma_L^A, \Gamma^{RS}$.

which satisfy the following sets of conditions (for the full, detailed list, see the appendix):

1. Given values, belief updates, and prices, all policy functions are optimal.

[3]For example, in the Greek Government Debt Restructuring of 2012, short term notes guaranteed by the EFSF were about a third of the portfolio lenders received in exchange for their existing bonds (Zettelmeyer et al., 2013). Since these notes were extremely safe and liquid, they are effectively a cash transfer.

2. Given prices, belief updates, and policies, all value functions satisfy their functional equations.

3. Given belief updates and policies, all price functions satisfy their functional equations.

4. All belief update functions are consistent with the policy functions and Bayes's Law.

3.5 Existence

In this section, I prove that, under certain assumptions, an equilibrium must exist. Some of these assumptions are simply to ease the notational burden of the proof and do not affect its generality. Others have a material impact on its generality. Whenever one falls in this second category, I note it explicitly. I begin by stating the assumptions I need to make about functional forms, distributions, and state spaces. Then I state the main theorem on existence, and sketch its proof (the full details are in the appendix).

Assumption 1. ***Finiteness of choice sets:*** *$\mathscr{B} \subset \mathbb{R}$ and $\mathscr{Q} \subset R$ are both nonempty and satisfy $|\mathscr{B}| = N_b < \infty$ and $|\mathscr{Q}| = N_Q < \infty$. $0 \in \mathscr{Q}$ and $\min\{\mathscr{B}\} = 0$.*

In short, choice sets facing all parties are always finite. $0 \in \mathscr{Q}$ guarantees that there is always a feasible choice for the government during the renegotiation process,[4] and the assumption that 0 is the lowest debt value eases the notation needed to define auction revenue.

Assumption 2. ***Preference shock distributions:***

1. *The preference shocks under repayment are distributed Generalized Type One Extreme Value with scale parameter σ_ϵ and correlation parameter ρ_ϵ.*

2. *The preference shocks while in default are distributed Type One Extreme Value with scale parameters $\sigma^Y_{\eta,X}$ where $X \in \{G, L\}$ and $Y \in \{P, R\}$.*

3. *The preference shocks while completing the restructuring process are distributed Type One Extreme Value with scale parameter σ^{RS}_ν.*

4. *The location parameters of every distribution are set such that the mean value of each individual shock is zero (i.e. $\mu = -\gamma\sigma$ where γ is the Euler-Mascheroni constant).*

[4] I could instead let the government decline to make an offer, but that would complicate the exposition.

The first key implication of this assumption are that ex ante values, choice probabilities, and posterior beliefs are continuous in the values of individual choices. The second is that, as the value of a choice goes to $-\infty$, the probability it is chosen goes to 0. The third is that, as the difference in value between two choices goes to $+\infty$, the ratio of their choice probabilities goes to $+\infty$. These second and third results allow me to show that there is an assignment rule for beliefs at infeasible choices that preserves the continuity of the update operator defined later. The Type One Extreme Value Shocks guarantee both of these as well as conveniently yielding analytical expressions for choice probabilities and ex ante values.

Assumption 3. ***Finiteness of state space:***

1. *$s \in \mathscr{S}$ with $\mathscr{S}$ nonempty and finite.*
2. *$T \in \{H, L\}$ and the transition matrix for T has entries $p_{HH} \in (0,1)$ and $p_{LL} \in (0,1)$ on its main diagonal. $\beta_H > \beta_L$.*
3. *$\pi \in \Pi$ with $\Pi \subset [0,1]$, $|\Pi| < +\infty$, and Π satisfying $\{0,1\} \subset \Pi$.*

Part 2 of this assumption eases the notational burden in the proof. Parts 1 and 3 of this assumption slightly reduce generality of the existence proof. They are required to ensure that the set of equilibrium objects can be construed as a single vector in a high but finite dimensional Euclidean space, so that I can apply results about continuous mappings between such spaces. Working in a similar environment, Chatterjee et al. (2020) make the same assumptions on the cardinality of $\mathscr{S}$ and Π. In order to deal with how beliefs evolve from one period to the next when the posterior belief at the end of a period $\hat{\pi}$ would evolve to a value $\hat{\pi}' \notin \Pi$, I follow Chatterjee et al. (2020) in defining the randomization rule $g(\pi', |\hat{\pi}')$ and modified expectation operator $\hat{\mathbb{E}}[.|\hat{\pi}']$ as follows:

Definition 1. ***Randomization rule for belief evolution:*** *For any function $f(\pi')$, set $\hat{\mathbb{E}}[.|\hat{\pi}']$ by:*

$$\hat{\mathbb{E}}[f(\pi')|\hat{\pi}'] = \sum_{\pi' \in \Pi} g(\pi'|\hat{\pi}') f(\pi')$$

So g maps $\pi \in [0,1]$ to weights on grid points in Π. I assume the following about g:

Assumption 4. ***Randomization function properties:***

1. *For every $\hat{\pi}'$, $\hat{\mathbb{E}}[\pi'|\hat{\pi}'] = \hat{\pi}'$.*

2. *For every $\pi' \in \Pi$ $g(\pi'|\hat{\pi}')$ is continuous in $\hat{\pi}'$.*

These simply require that g maintain the consistency of beliefs, on average, as well as guaranteeing that certain key expected value terms be continuous in posterior beliefs. Finally, I make some assumptions about the issuance cost function and then the utility function.

Assumption 5. ***Functional form of issuance cost:*** *$i(s, \pi, b, h, b')$ can be expressed as:*

$$i(s, \pi, b, h, b') = \begin{cases} 0 & b' < (1-\lambda)b \\ \hat{i}(\delta(s, \hat{\pi}', b'))q(s, \hat{\pi}', b')(b' - (1-\lambda)b) & b' \geq (1-\lambda)b \end{cases}$$

where $\hat{\pi}'$ is the next period prior after beliefs are updated from π, taking into account other previously observed actions this period h as well as the debt choice b', $\delta(s, \hat{\pi}', b')$ is the probability of a default occurring in the next period, and $\hat{i} : [0,1] \to [0,1]$ is a continuous function.

In short, the cost of issuing debt is a fraction of the resulting revenue is a continuous function of the next period default probability, and depends on nothing else. I make the following assumptions about the utility function and the costs of defaulting:

Assumption 6. ***Utility function:*** *$u : \mathbb{R}_{++} \to \mathbb{R}$ is continuous, increasing, and has $\lim_{x \downarrow 0} u(x) = -\infty$.*

These are standard conditions. I can now state the main theorem of this section

Theorem 1. ***Equilibrium Existence:*** *Suppose that assumptions 1, 2, 3, 4, 5, and 6 hold. Then an equilibrium exists.*

Proof: see appendix.

While I relegate the full details of the proof to the appendix, I sketch its main components here. The strategy is partially based on the existence proof in Chatterjee et al. (2020). Finiteness of the (s, π, b) states and the choice sets guarantees that there are only finitely many combinations of states and within-period sequences of actions. Thus, all values, prices, and belief updates are vectors in some high (but finite) dimensional Euclidean Space. This gives me access to certain theoretical results about continuous mappings between compact, convex sets of such spaces (in particular, Brouwer's Fixed Point Theorem). The assumptions on the issuance cost function and the randomization rule guarantee that consumption values are continuous in prices and posterior beliefs, and the parametrizations of the preference shocks ensure that both choice probabilities and ex ante values are continuous functions of the values associated with the individual choices. This ensures continuity of the mapping which updates the values of all equilibrium objects at any feasible choice sequence (where feasibility means that the consumption value yielded by a choice sequence is strictly positive).

The main complication that arises in the proof is choosing how beliefs evolve after infeasible choices to ensure that, as a choice becomes just infeasible (i.e. $c \downarrow 0$), the limit of the sequence of posterior beliefs matches the value assigned when that choice is infeasible. While it is easy to show that values and choice probabilities remain continuous here, the definition for the posterior belief reads $\frac{0}{0}$ in the limit. It turns out that the consistent assignment rule for posterior beliefs after an infeasible choice puts full weight on type with the highest β. While all types play the action that is just barely feasible very rarely, the type with the highest beta puts the smallest weight on the extremely large negative number associated with utility from consumption in the current period, since values are written given by:

$$(1 - \beta_T)u(c) + \beta_T EV(T)$$

Therefore, as $u(c)$ goes to $-\infty$, the type with the highest β chooses this option infinitely more frequently than the other types, ensuring that the limit of the posterior beliefs puts full weight on the type with the highest β, just like the assignment rule at infeasible choices.

4 Calibration

In this section, I describe the patterns in the data that identify the type- and reputation-related parameters of the model. Then I describe the functional forms I use in the quantitative implementation of the model and detail the calibrated parameter values. After that, I show how well the models fits the data.

4.1 Identification

After completing negotiations with creditors and exiting default, countries that have defaulted pay higher interest rates than appear to be justified by their debt levels and the state of their economies. There are many empirical papers that verify that a regression of interest rate spreads on economic, political, and other relevant observables, as well as credit history variables, will yield a set of jointly significant effects for the credit history variables. Specifically, the coefficients α_τ on dummy variables $d_{it,\tau}$ indicating that at time t, country i defaulted (or restructured) τ years ago in the specification below will be significant:

$$spread_{it} = X_{it}\beta + \sum_{\tau \in \mathscr{T}} \alpha_\tau d_{it,\tau} + \epsilon_{it}$$

In general, they have positive signs and are declining in magnitude as τ rises (i.e. the effect of the average default on spreads fades over time). Furthermore, the data show that it is not just the extensive margin of default vs. repayment which matters for this effect. Rather, the intensive margin of how severely lenders suffered in the restructuring is correlated with its scale. Cruces and Trebesch (2013) show that under a wide variety of ways of measuring investor losses, often called "haircuts," h_{it} (with $\bar{h}$ their mean), the slope coefficients γ_τ in the augmented specification below will also be jointly significant:

$$spread_{it} = X_{it}\beta + \sum_{\tau \in \mathscr{T}} d_{it,\tau}(\alpha_\tau + \gamma_\tau(h_{it} - \bar{h})) + \epsilon_{it}$$

In their work, the average effects α_τ are positive and statistically in the first two years after a default (in the range of $150 - 300$ b.p.), but quickly fall to zero thereafter. On the other hand, the estimated marginal effects γ_τ are close to zero in the first two to three years, but become positive and statistically significant thereafter. I use these two patterns, together, to identify the key parameters controlling the stochastic type process in my model.

The patterns of behavior in the model differ qualitatively based on the pairing of the differences between the two types (whether the less patient type finds default less costly or more costly). For this reason, I focus here on how the data identify that pairing in the context of my model. Both pairings can generate the first pattern (α_t positive in the first few years and approximately zero thereafter). Only one of them can deliver the second.

To understand why that is the case, it is useful to precisely define the two in the context of my model. The α_t, for $t \in \{1, ..., N\}$, measure differences in interest rates between countries that restructured t years ago and ones that have not restructured in at least $N+1$ periods ("non-restructurers"), conditional on observables (s, b'). The γ_t, on the other hand, measure the correlation between haircut and differences in interest rates between restructurers, conditional on on observables (t, s, b').

Both β_T and $\hat{\phi}_T$ are important, first order determinants of how frequently type T ends up defaulting (and therefore having to restructure). β_T controls the rate at which they accumulate debt up to levels where default may occur, and $\hat{\phi}_T$ determines how high such levels are. Furthermore, the type-specific default cost $\hat{\phi}_T$ is a key, first order determinant of the government's outside option during the renegotiation process. A type with a higher default cost $\hat{\phi}_T$ has a worse outside option and will, all else equal, extract less of the surplus from making a deal with lenders and therefore get a lower haircut.

The estimated γ_t being close to 0 in the first two years means that knowing the haircut does not provide much additional information about the country's interest rate beyond what is contained in the observables (t, s, b'). After the first two years, however, the γ_t rise away from 0 and become statistically significant. This means that, once a couple of years have passed, knowing what the haircut was at the time of the restructuring does provide significant, additional information about the interest rate beyond what is contained in (t, s, b'). This pattern is possible only if the type that is more patient finds default more costly.

Call the type that finds default more costly the "high" type. When this type is more patient than the "low type" ($\hat{\phi}_H > \hat{\phi}_L$ and $\beta_H > \beta_L$), then the high type can support more debt than the low type but has a weaker desire to borrow. After a restructuring, the two types may separate in the b dimension, as the low type borrows more quickly. During that initial separation, (t, s, b') sharply identify type, and haircut does not provide much more information. However, the low type will soon reach its borrowing "limit" (i.e. levels of debt beyond which default becomes likely) and slow down or stop accumulating debt. Now, relative to international investors, the high type is impatient (just less so than the low type), and therefore does want to borrow. Furthermore, because it can support more debt, it is willing to borrow out to a higher "limit". Thus it will eventually "catch up" to the low type in the debt dimension. At this point, the observables (t, s, b') leave a lot of ambiguity about the country's type–some of which can be resolved using knowledge of the haircut.

If, on the other hand, the high type is less patient than the low type ($\hat{\phi}_H > \hat{\phi}_L$ and $\beta_H < \beta_L$), then the high type can support more debt than the low type and has a greater desire to borrow. Then, after a restructuring, the two types will separate in the borrowing dimension. The high type wants to borrow quickly and is able to borrow to higher levels of debt, and the low type will never catch up. Therefore, once a few years have passed, knowing b' as well as the time since restructuring t and the current state s should be a near perfect indicator of the country's type. Thus, the additional information contained in the haircut would fall to 0 within a few years. Since this is not the case in the data, I can rule out this pairing.

Given the signs of $\beta_H - \beta_L$ and $\hat{\phi}_H - \hat{\phi}_L$, the magnitudes of the effects in the data help identify the magnitudes of the differences in preferences. In addition to these specific differences in preferences, there are several other pieces of the model that these patterns help identify. The persistence of the types is closely associated with how long lasting the effects are. The preference shock parameters associated with the renegotiation and restructuring process help govern the flow of information when the government is in default. Conditional on specific differences in preferences, these therefore govern how precise (or imprecise) beliefs are when a restructuring is completed, which moderates how much the differences in preferences are translated into the reduced form effects on prices observed in the data. Finally, the distribution of the preference shocks during repayment help control the flow of information when the country is in good standing, which affects how much more (or less) accurate beliefs become in the years following a restructuring, which affects the trends of the effects.

4.2 Functional Forms and Parameters

The model is calibrated to match the experience of Argentina since 1993. The quarterly risk-free real interest rate, r is set to 0.01, a standard value. The maturity rate λ of the bond and its coupon value κ are set to the values used by Chatterjee and Eyigungor (2012) (who also study Argentina during mostly the same period). I also use their parameter estimates for the income process (assumed to be AR(1)), $\rho_y = 0.95$ and $\sigma_y = 0.03$. The functional form of utility was assumed to be constant relative risk aversion:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

The relative risk aversion coefficient γ is set to 2, a standard value in macroeconomics. Table 1 summarizes the parameters set outside the model.

Table 1: Parameters Set Independently

Parameter	**Value**	**Source**
γ	2.00	Standard
r	0.01	
λ	0.05	Chatterjee & Eyigungor (2012)
κ	0.03	
ρ_y	0.95	
σ_y	0.03	

Other functional forms that I must specify are the flow utility cost of default and the issuance cost function. I use a flow utility cost of default, rather than an output cost of default (which is used by Arellano (2008) and Chatterjee and Eyigungor (2012), among many others), in order to avoid the realization of output while in default perfectly communicating the government's type to lenders. Note that this assumption does not imply that there are no real output costs of default, just that those costs are felt differently by the two government types. In order to make clear the relationship between the default costs in my model and those employed in the literature, I define the utility cost of default implicitly by:

$$u(y(s)) - \phi_T(s, d_t) := u\Big(y(s) - \max\{(h_0 + \tilde{\phi}_{d_t} + \hat{\phi}_T)y(s) + h_1 y(s)^2, 0\}\Big)$$

One of the most common parametrizations of the cost of default in the sovereign default literature is a linear-quadratic cost in output (see Chatterjee and Eyigungor (2012), for example). The functional form replicates that. Furthermore, it allows for both 1) a constant percent difference in the costs for the two types, and 2) a constant percent difference between the cost of triggering default and the cost of remaining in default (specified by $\tilde{\phi}_{d_t}$).

The issuance cost function is assumed to have the following form:

$$i(s,b,\pi',b') = \begin{cases} 0 & b' \leq \hat{b} \text{ or } Pr(d'^{\star}=1) \leq p_d \\ q(s,\pi',b')(b'-\hat{b})\hat{i}(s,\pi',b') & b' > \hat{b} \text{ and } Pr(d'^{\star}=1) > p_d \end{cases}$$

where $\hat{b} = \max\{(1-\lambda)b, 0\}$.[5] The purpose of issuance cost functions in this type of model are to prevent a behavior Chatterjee and Eyigungor (2015) termed "maximum dilution." Essentially, one period before default, the maturity structure of the debt gives the government an incentive to issue an enormous amount of debt, completely extracting the value of existing bondholders' securities. Issuance cost functions counteract these incentives.

The distribution of the preference shocks during repayment is assumed to be Generalized Type 1 Extreme Value. The distributions of all the other preference shocks in the model are assumed to be Type 1 Extreme Value. These distributions are chosen for their computational tractability. Specifically, both choice probabilities and ex ante expected values can be written analytically in terms of the values associated with the choices (McFadden, 1978).

Apart from the parameters specified in Table 1, all parameters are calibrated by simulated method of moments. The targeted moments are the mean and volatility of the external debt to GDP ratio while not in default, the mean and volatility of spreads while not in default, the default rate, the average haircut, the average delay between a default and a restructuring, and the average rise in rise in the debt to GDP ratio in the one year preceding a default, as well as the five average effects α_τ and five marginal effects γ_τ from the regression of Cruces

[5] The specific functional form of $\hat{i}(s,\pi',b')$ is given by:

$$\hat{i}(s,\pi',b') = \frac{1}{2}\left(1+\sin\left(\pi\Big(\frac{Pr(d'^{\star}=1)-p_d}{1-p_d}-\frac{1}{2}\Big)\right)\right)$$

i.e. a sine wave shifted and scaled to rise from 0 to 1 as it travels from p_d to 1. This function combines elements of two main types of issuance cost functions used in the literature. The first, used by Chatterjee and Eyigungor (2015), as well as others, is a strict limit on the one period ahead default probability (or spread) (i.e. cost is 0 up until some value and then infinite thereafter). The second, used by Dvorkin et al. (2021), as well as others, is a continuous, convex cost in the scale of the issuance. $\hat{i}(s,\pi',b')$ combines the 0-up-to-a-threshold property of the the first class of functions with the continuity (and some of the convexity, at least for lower values of $Pr(d'^{\star}=1)$) of the second.

and Trebesch (2013):

$$spread_{it} = X_{it}\beta + \sum_{\tau \in \mathscr{T}} d_{it,\tau}(\alpha_\tau + \gamma_\tau(h_{it} - \bar{h})) + \epsilon_{it}$$

The target mean and volatility of external debt to GDP were calculated using annual Argentinian data from 1993 to 2019 from the World Bank's International Debt Statistics (IDS) (formerly Global Development Finance data), excluding years in default. The target mean and volatility of spreads were calculated based on monthly Argentinian EMBIG spreads from 1997 to 2019, excluding months in default, from the World Bank's Global Economic Monitor (GEM). The target default rate was taken from Chatterjee and Eyigungor (2012).

Due to the relative rarity of restructuring events, the remaining targets were calculated using cross-country evidence. The target average haircut and average delay were calculated using data from Cruces and Trebesch (2013) and Asonuma and Trebesch (2016), excluding donor-funded restructurings. The target average rise in debt to GDP in the year prior to a default was calculated based on data presented in Benjamin and Wright (2013). The target values for the regression coefficients were taken directly from Cruces and Trebesch (2013).

While all of the parameters affect all moments and are jointly calibrated, I will provide some insight, when I can, into which parameters are identified by which moments. The first five parameters are $(\mathbb{E}[\beta_T], \mathbb{E}[h_0 + \hat{\phi}_T], h_1, \sigma_\epsilon, \rho_\epsilon)$. These govern the average impatience of the government, the average penalty for defaulting, and the distribution of the preference shocks under repayment (σ_ϵ is the scale parameter for Generalized Type 1 Extreme Value distribution of preference shocks under repayment, and ρ_ϵ is its correlation parameter (for the repayment nest)). These parameters are closely tied to the means and volatilities of the debt to GDP ratio B'/Y and interest rate spreads $r - r^f$, as well as the default rate. As mentioned earlier, σ_ϵ and ρ_ϵ also effect the patterns in the α_τ and γ_τ as τ changes.

The next three parameters are (p_d, ψ, μ_G). These govern the shape of the issuance cost function, the frequency at which renegotiation opportunities arise, and the bargaining power of the government during renegotiation. p_d, the threshold for default probability at which the issuance cost begins kicking in, is tightly tied to the rise in debt over the one year preceding a default. The other two parameters in this group, ψ and μ_G, control two key characteristics of the renegotiation process and are informed by data on restructuring outcomes. The value of ψ is closely tied to the delay between default and the completion of the restructuring process. The parameter μ_G governs the bargaining power of the government during the renegotiation process and is therefore closely linked to the average haircut imposed on lenders.

The last ten parameters are $(p_{HH}, p_{LL}, \beta_H - \beta_L, \hat{\phi}_H - \hat{\phi}_L, \tilde{\phi}_1, \sigma_\eta^{P,G}, \sigma_\eta^{R,G}, \sigma_\eta^{P,L}, \sigma_\eta^{R,L}, \sigma_\nu^{RS})$. The first two, p_{HH} and p_{LL}, are the probability of remaining the high type and the probability of remaining the low type. The next two, $\beta_H - \beta_L$ and $\hat{\phi}_H - \hat{\phi}_L$ are the difference between the impatience rates of the two types and the difference between their as-if output costs of default. $\tilde{\phi}_1$ controls the extra penalty associated with the initial period of default, and is closely associated with the average haircut. $\tilde{\phi}_0$ is normalized to 0. The final five are preference shock parameters associated with the renegotiation process. $\sigma_\eta^{X,Y}$ is the scale parameter for the Type 1 Extreme Value preference shocks for party X when that party has role Y in the renegotiation process (so, for example, $\sigma_\eta^{P,G}$ refers to the government's preference shocks when it proposes a deal). The final parameter in this block, σ_ν^{RS}, is the scale parameter for the preference shocks in the restructuring decision problem of the government. As discussed in the identification section, these are identified by regression coefficients α_τ and γ_τ.

The model was solved in Julia using discrete state space methods. For details, see the appendix. The full set of parameters calibrated jointly is detailed in table 2 below:

Table 2: Calibrated Parameters

Parameter	**Value**	**Parameter**	**Value**
$\mathbb{E}[\beta_T]$	0.947	σ_ϵ	$1.7e-4$
$\mathbb{E}[h_0 + \hat{\phi}_T]$	-0.159	ρ_ϵ	0.378
h_1	0.219	$\sigma_{\eta,G}^P$	$6.7e-4$
p_{HH}	0.986	$\sigma_{\eta,L}^P$	$3.2e-3$
p_{LL}	0.984	$\sigma_{\eta,G}^R$	$1.8e-4$
$\beta_H - \beta_L$	0.043	$\sigma_{\eta,L}^R$	$1.4e-2$
$\hat{\phi}_H - \hat{\phi}_L$	0.022	σ_ν	$2.2e-3$
p_d	0.322	ψ	0.080
μ_G	0.906	$\tilde{\phi}_1$	0.136

The calibrated average impatience is very close to what is the calibrated impatience used in other work on sovereign default in emerging market economies (for example, Chatterjee and Eyigungor (2012) estimate $\beta = 0.954$). The average penalty scale is also pretty similar to their calibration ($h_0 = -0.19, h_1 = 0.25$). Since both models are calibrated to the Argentinian economy, this should not come as a surprise.

The most interesting features of the calibration are the parameters describing the two types (highlighted in red in Table 2). Both the high and low type are quite persistent, with the high type lasting 17.5 years on average and the low type lasting 15.5 years on average. Since the high type is slightly more persistent, time in power is split 53% to 47% in its favor. There is also a relatively large difference in how impatient the two types are, with the difference in discount factors over 4%. In fact, the discount factor of the high type $\beta_H = 0.967$ is actually closer to the lender discount factor of $\frac{1}{R} = 0.990$ than it is to the discount factor of the low type $\beta_L = 0.924$. There is a somewhat smaller difference in how painful the two types find default. In particular, the high type finds default (in terms of the as-if output cost) just 2.2 percentage points more painful. For reference, the cost for the low type at the mean level of output is 6.0% (after the initial period of default).

4.3 Targeted Moments

Let me now discuss how the model fits the data. Data and model values for the non-regression coefficient targeted moments are detailed in Table 3 below:

Table 3: Targeted Moments (Annualized Values)

Group	Moment	Data	Model
1	$\mathbb{E}[B'/Y]$	21.89%	19.44%
	$\sigma(B'/Y)$	6.19%	5.44%
	$\mathbb{E}[r - r^f]$	7.57%	5.80%
	$\sigma(r - r^f)$	4.71%	6.55%
	$\mathbb{E}[d]$	12.50%	12.55%
2	$\Delta_1(B'/Y \mid d = 1)$	6.0 p.p.	5.0 p.p.
	$\mathbb{E}[\text{delay}]$	3.23	3.30
	$\mathbb{E}[h]$	29.73%	31.87%

Group 1 contains moments calculated using only data from Argentina. Group 2 contains moments calculated based on data from a large sample of countries. Figures 1 and 2 below show the model fit of the regression coefficients visually:

Figure 1: Model Fit: Average Effects

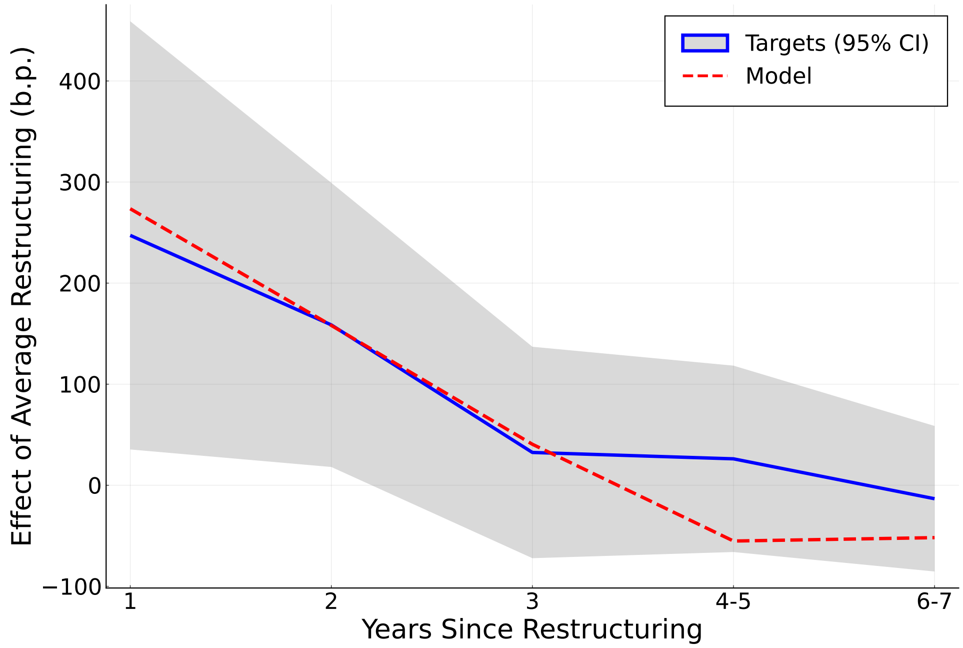
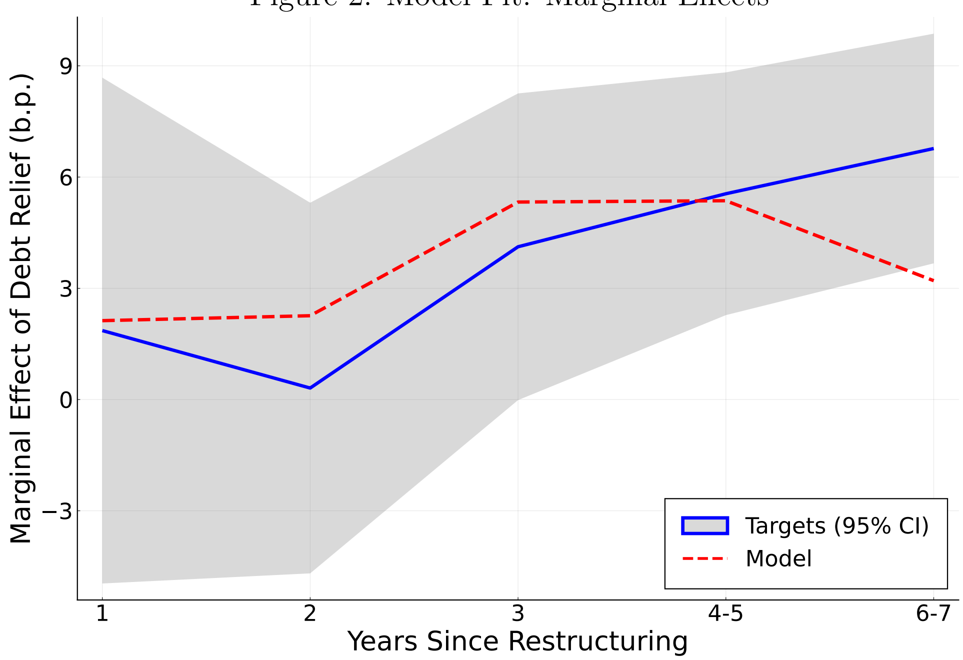


Figure 2: Model Fit: Marginal Effects



In the first two blocks, there are only two medium size misses: the mean and volatility of spreads. Since the model lacks risk premia, it is unsurprising that it struggles to match the full average level of spreads in the data. Among the regression coefficients, only one miss is statistically significant: the slope coefficient on the haircut effect after $6-7$ years. This is caused by targeting a significantly higher default rate than the average in the sample for the regression, resulting in quicker attrition of the riskier members of the sample.

5 Results

5.1 Validation

Now that I have described the data that identify my the key parameters of my model and my overall calibration strategy, I move on to a section validating my approach. Specifically, I will use the model to develop a method to use real world data on debt issuance to measure reputation. I then show that this model-implied measure has real quantitative bite.

This model was fit by matching correlations between historical default and restructuring choices, and current interest rate spreads. This is all based, of course, on post-default facts. However, the model also makes a rich set of predictions about how debt issuance behavior, conditional on not defaulting, should affect reputation and therefore spreads. In this section, I check whether these predictions are borne out in the data.

To do this, I first compress all the model's belief update functions into a parsimonious functional form that only depends on current period reputation and current period gross issuances of debt divided by GDP GI. This focus on debt issuance is motivated by the fact that, as discussed in the next section, that borrowing decisions are one of the most important channels by which information is conveyed to lenders.[6] Furthermore, the relationship between debt issuances and reputation was not directly targeted when fitting the model. To obtain my parsimonious approximation of the belief update function, I estimate the following regression equation using simulated data:

$$ln\left(\frac{\pi_t}{1-\pi_t}\right) = \beta_0 + \beta_1 ln\left(\frac{\pi_{t-1}}{1-\pi_{t-1}}\right) + \beta_2 GI_t + \beta_3 ln\left(\frac{\pi_{t-1}}{1-\pi_{t-1}}\right) \beta_3 GI_t + \epsilon_t$$

[6]This result is discussed and explained at length in the next section of the paper.

This resulting approximate belief update equation is very easy to take to the data and is actually pretty accurate in model simulated data (the R^2 is about 82%). With the estimated coefficients $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2, \hat{\beta}_3, \hat{\beta}_4)$ in hand, I then proceed to the main exercises of this section. I use IDS data on debt issuance and GDP (augmented with OECD data for a few countries) to produce full sequences of model-filtered reputation for most of the countries in the Cruces and Trebesch (2013) sample. I then estimate this augmented version of their specification:

$$spread_{it} = X_{it}\beta + \sum_{\tau \in \mathscr{T}} d_{it,\tau}(\alpha_\tau + \gamma_\tau(h_{it} - \bar{h})) + \beta_\pi \hat{\pi}_{it} + \epsilon_{it}$$

The results of this estimation (and an alternative version where I add gross issuances of debt divided by GDP instead of filtered reputation) are detailed below in Table 4:

Table 4: Regression Results: Interest Rate Spreads

	EMBIG Spread		
$\hat{\pi}$	–	-192^{***} (67)	–
GI	–	–	1047^{*} (599)
Debt to GDP (%)	8.0^{***} (2.5)	6.3^{**} (2.5)	7.0^{***} (2.6)
Other Controls	Yes	Yes	Yes
Country & Year Fixed Effects	Yes	Yes	Yes
R^2	0.3353	0.3421	0.3378
Observations	3378	3378	3378

These regression results show that the model's filtered measure of reputation provides significant additional explanatory power when compared to the reference specification. The fact that current gross issuances alone do not pass the same test show that this is not just due to the fact that reputation incorporates them. The way the model aggregates the history of gross issuances into a single term is providing significant additional information. The effect of reputation has the predicted sign and is economically significant in magnitude. The difference in spreads between two otherwise identical countries, one with the worst possible reputation and one with the best possible reputation, is about two percentage points.

Finally, I evaluate how well this filtered measure of reputation predicts future defaults.

Specifically, I estimate the following logit model with fixed effects:

$$\log\left(\frac{p_{it}}{1-p_{it}}\right) = X_{it}\beta + \sum_{\tau\in\mathscr{T}} d_{it,\tau}\alpha_\tau + \beta_\pi \hat{\pi}_{it}$$

where p_{it} is the probability of defaulting within the next year.[7] The results of three regressions specifications are detailed below in Table 5:

Table 5: Regression Results: Default Probability

	Default within the Next Year		
$\hat{\pi}$	–	-35^{***} (9)	–
GI	–	–	-61^{**} (28)
Debt to GDP (%)	0.65^{***} (0.15)	-0.37^{**} (0.19)	0.92^{***} (0.21)
Other Controls	Yes	Yes	Yes
Country Fixed Effects	Yes	Yes	Yes
LL	-97	-54	-95
Observations	1065	1065	1065

I have included the estimated coefficient on debt to GDP in this table in order to highlight how it changes when different sets of covariates are used. In particular, the effect of debt to GDP actually becomes negative (albeit with a p-value of 0.048) when filtered reputation is included in the specification! Filtered reputation is highly significant and dramatically increases the fit of the regression. While current period gross issuances alone is significant and slightly improves the fit of the regression, it does not add nearly as much information as the full history of debt issuances, aggregated through the lens of my model.

Table 6 contains a selection of untargeted moments. In particular, it contains data and model values for the first and second moments of debt to GDP and interest rate spreads when the sample is not restricted to periods out of default. The model slightly underestimates long run levels of debt and significantly underestimates the long run volatility of debt to GDP. On the other hand, the model's estimates of the mean long run spread and the volatility of long run spreads are very close to their data counterparts.

[7] I have omitted the intensive margin effects of haircuts from this specification because, within the reduced sample that the fixed effects logit model requires, they are extremely collinear with the extensive margin dummies and the estimation fails to converge when they are included.

Table 6: Other Moments

Moment	Data	Model
$\mathbb{E}[B'/Y]$ (including while in def.)	28.95%	23.6%
$\sigma(B'/Y)$ (including while in def.)	19.76%	7.1%
$\mathbb{E}[r - r^f]$ (including while in def.)	15.14%	13.1%
$\sigma(r - r^f)$ (including while in def.)	17.42%	12.9%
$\rho(r_t - r^f, r_{t-1} - r^f)$ (including while in def.)	0.92	0.95
% of defaults with $Y < E[Y]$	61%	61%
Effect of q_0^D on haircut	-0.60	-1.35

The final two entries in this table are the autocorrelation of interest rate spreads, the percent of defaults occurring when output is below trend, and the estimated effect on future haircuts of the price of a bond that has just been defaulted on, in a linear regression. The model slightly overestimates the autocorrelation of spreads. Furthermore, it manages to match the fact that a small majority of defaults occur when default is below trend. The specific definition of the final moment is δ_1 in the regression:

$$h_j = \delta_0 + \delta_1 q_{0,j}^D + e_j$$

where h_j is the haircut observed in restructuring j and $q_{0,j}^D$ is the average price of the bonds restructured, measured one month after the default occurred.[8] Meyer et al. (2021) report a highly significant value of -0.60 for the effect δ_1 and use this result to argue that bond prices directly after default forecast haircuts in the eventual restructurings (possibly years away) rather well. This relationship is nontrivial because haircuts measure the ex post difference between the value of old debt and the value of new debt (a relative quantity), while the bond price right after default measures the actual ex ante value of the debt (an absolute quantity). My model replicates the sign of this relationship, but the association is somewhat sharper. To my knowledge, my paper is the first to replicate this pattern. While the first five data values in this table were calculated using only data from Argentina, the data values for the sixth and seventh untargeted moments reported in this table were calculated by Benjamin

[8] I map $q_{0,j}^D$ to the price of the bond immediately after the government makes its default decision (and before the resolution of any uncertainty about whether there will be an opportunity to renegotiate in the current period). Since the quarterly probability of such an opportunity arising is relatively small, this result is robust to other assumptions about the timing of this measurement.

and Wright (2013) and Meyer et al. (2021), respectively, using cross country samples.

5.2 Why Do Defaulters Have Bad Reputations?

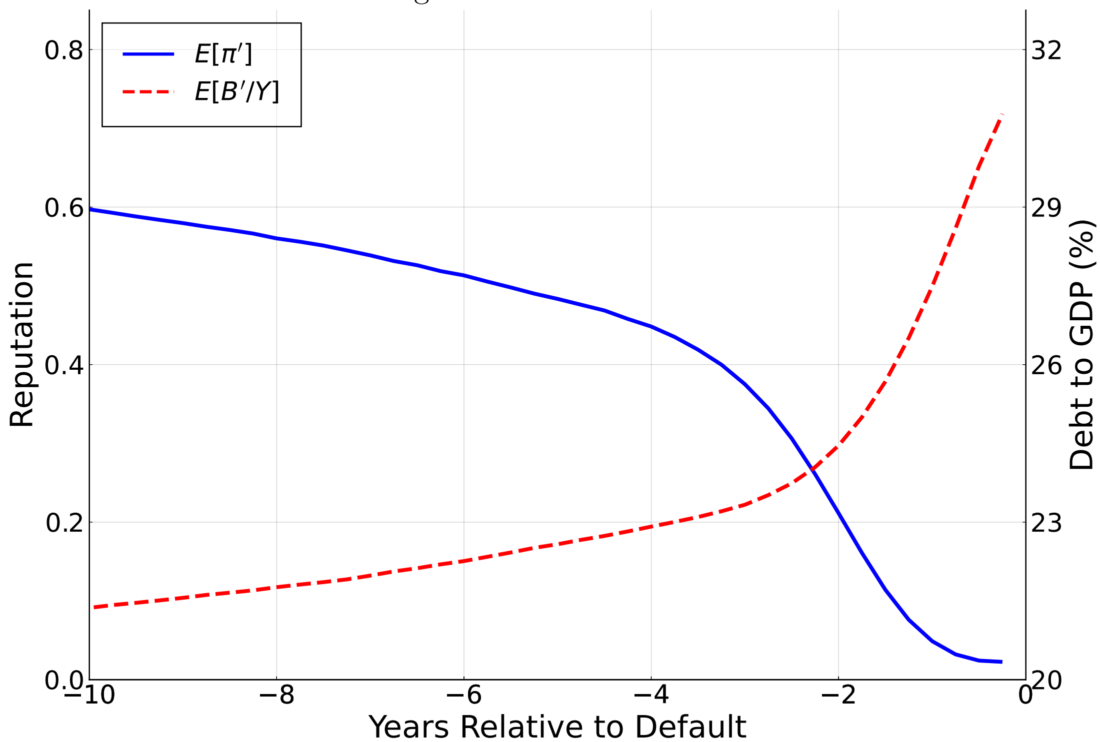
Having established that my approach to modelling sovereign borrowing with reputational concerns is validated by the data, I now return to one of the key questions I set out to answer, "Why do defaulters have bad reputations?" The most intuitive interpretation of the data we have is that defaulting wrecks a country's reputation, but there are other stories that are just as consistent with the data. It could also be the case that having a bad reputation causes countries to default because it makes it very expensive to roll over debts. Or it could be that merely accumulating enough debt that default may occur with non-trivial probability wrecks a country's reputation. These three stories have very different implications for a wide variety of questions but are all, on their face, equally consistent with the data we have.

Using my model, I can distinguish between them. In the model, right after a default occurs, lenders' posterior belief that the country is the responsible type is 2.31%. Thus, the model delivers the pattern that countries which have recently defaulted have relatively poor reputations. However, this is not because the default decision itself revealed that the government were the irresponsible type. In fact, conditional on default occurring in the current period, the average beginning of period reputation value is 2.28%, quantitatively indistinguishable from the posterior. Instead, I find that all of the borrowing decisions required to run up the debt to the point where default was a nontrivial possibility were what destroyed the country's reputation in international markets. Figure 3 illustrates this pattern.

Figure 3 plots the the average end of period reputation π' and end of period debt to GDP ratio B'/Y during the 10 years prior to a default. Over these years, countries steadily increase their debt levels[9] at the expense of their reputation. Both sides of this accelerate three years before a default. One force driving this acceleration is that, as the country's reputation falls, it becomes more and more painful for the irresponsible type to convincingly imitate the responsible type. Eventually, mimicry becomes costly enough that the irresponsible type

[9]The pattern in Figure 3 is driven primarily by increases in the debt stock B' rather than decreases in output Y. If the picture is reproduced using $\mathbb{E}[B']$ instead of $\mathbb{E}[B'/Y]$, it looks extremely similar.

Figure 3: Pre-Default Paths



simply gives up entirely and just borrows more. This result, that reputations are lost before default, not by default, is one of the core contributions of my paper.

This also leads to a key implication of my paper for policy. In debates about whether to rescue a country from default, concerns about the reputational consequences of defaulting often play an outsize role. To determine how relevant such concerns are, I quantify the reputational benefits of repayment during crises. I define a "crisis" as any state where, before preference shocks are realized, lenders believe the risk of default is at least 1% (these results are robust to using alternative thresholds). The government's average reputation in such states is under 10%. To measure the value of the reputational benefit of repayment, I compare the government's value $V(s, T, \epsilon, \pi, b)$ to the following alternative value:

$$V^{Alt}(y, T, \epsilon, \pi, b) = \max_{d \in \{0,1\}} (1-d) V^R(y, T, \epsilon, \hat{\pi}^D, b) + d(V_0^D(y, T, \hat{\pi}^D, b) + \epsilon^D)$$

$$\hat{\pi}^D = \Gamma^D(1, \pi | y, b)$$

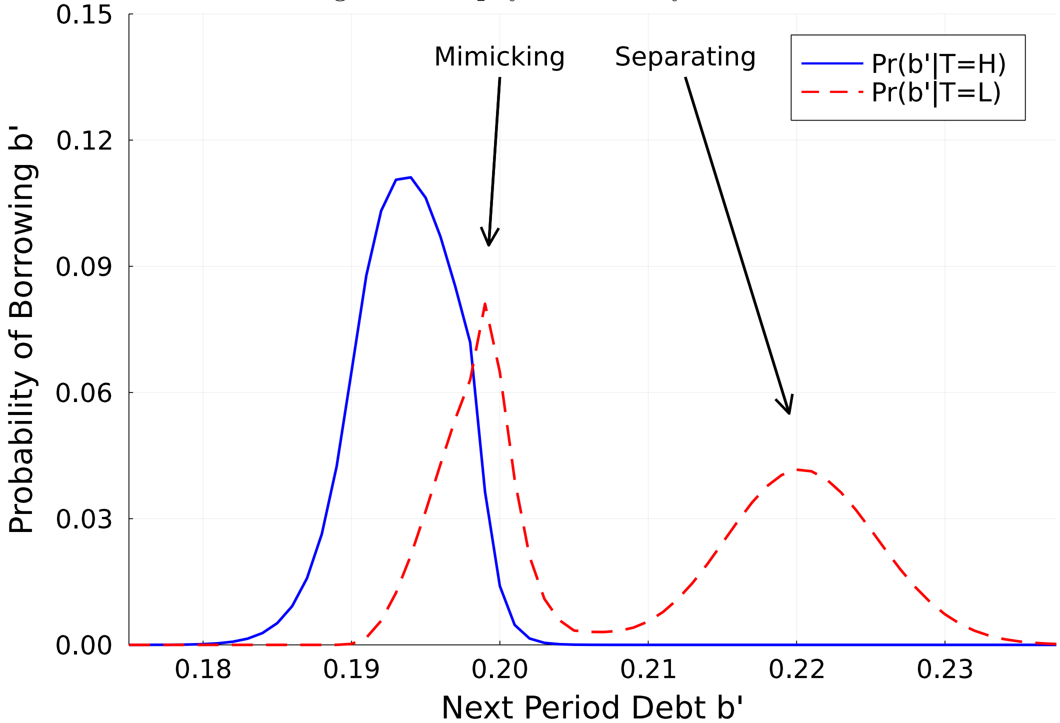
In this alternative, the government's reputation after choosing whether to default is fixed at the value associated with defaulting, regardless of whether or not the government actually chooses to default. Therefore, there is no reputational benefit of choosing to repay lenders instead of default. I then measure the associated welfare losses, which are just under 0.5 basis points of permanent consumption. In terms of current period consumption only, this is equivalent to a change of about 6 basis points. To put that in perspective, the current period consumption cost of repayment $\mathbb{E}[\frac{y-c}{c}|d=0]$ is just over 6%, over 100 times larger!

Therefore, in the vast majority of cases, reputational consequences of the default vs. repayment decision are not an important consideration for troubled countries. By the time they find themselves exposed to nontrivial levels of default risk, they have very low reputations, and what reputation can be gained by opting not to default is not particularly valuable. This is not, however, because the model never assigns high value to having a good reputation. In certain states of world, a good reputation can be worth up to 2% of permanent consumption. It is just that the value of having a good reputation is the ability to borrow at good prices. That ability is particularly valuable when the country has very low debt and more room to borrow, not when it has accumulated so much debt that it is on the edge of default.

5.3 Role of Asymmetric Information

I now describe the role of asymmetric information in the model overall as well as specifically in producing the above results. I begin by plotting a typical set of borrowing policy functions for the government in Figure 4. The b' axis here is b' divided by the average annual value of GDP, to make its values comparable to the moments reported above. These policies are generated by a level of debt close to the mean, the mean value of income, and a high value of beginning-of-period reputation. Here, the low type is torn between revealing itself entirely and borrowing relatively more, or preserving some of its reputation by choosing relatively lower borrowing levels that the high type also chooses frequently. The high type in turn knows that the low type chooses those borrowing levels sometimes and in turn tilts its choices even lower, signalling to lenders that it is almost certainly the responsible type. This signalling mechanism is the key to one of major roles of asymmetric information in this

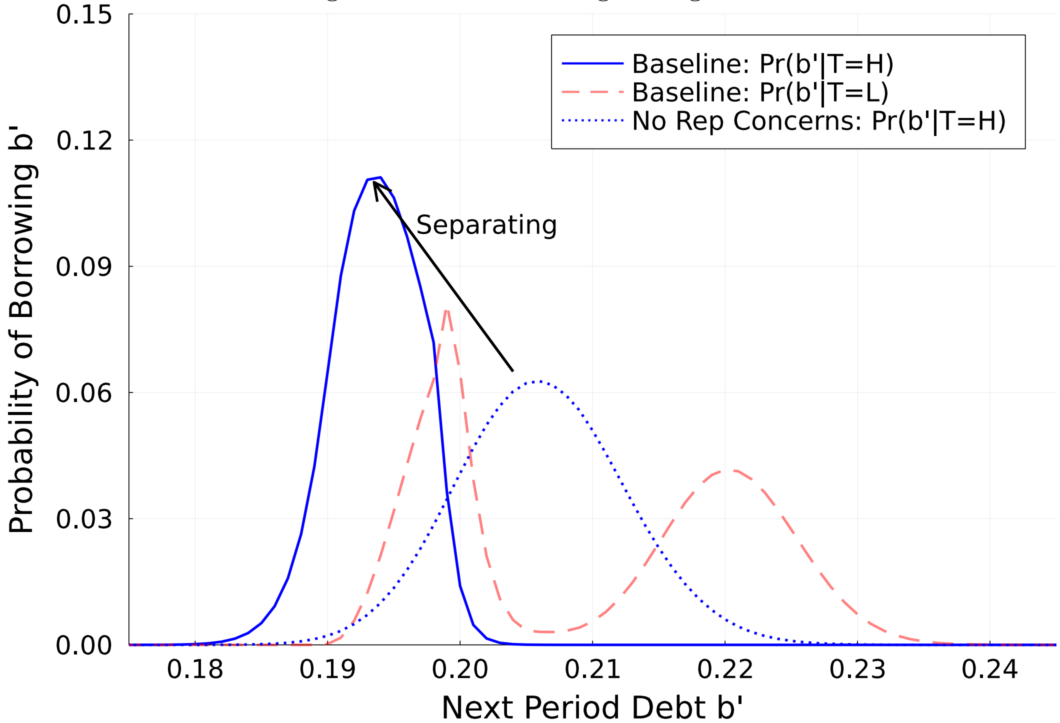
Figure 4: Repayment Policy Functions



model. The higher patience level of the high type provides an initial, basic incentive not to borrow as much as the low type. The signalling mechanism reinforces this by adding extra curvature to the price function faced by the government as it adjusts its debt level.

In order to illustrate exactly how much signalling motives distort the high type's choices, in the short run, I compare its equilibrium choices to what it would choose, were its type fully revealed for one period only, as a surprise (the shock is assumed to never repeat). When this surprise revelation occurs, signalling motives (in the current period) vanish. Figure 5 plots this alternative policy function, overlaid on the baseline policies from Figure 4. Figure 5 shows that if the high type did not have to worry about reputational consequences, it would choose to borrow substantially more than it does in the baseline. Furthermore, the strength of this effect is not unique to the point chosen for the example policy functions above. The average difference between the alternative policies and the baseline policies, under the long run joint distribution of states implied by the baseline model, is 0.80% of GDP when the government is the high type and 0.12% of GDP when the government is the low type. In the long run, the repeated effects of this signalling mechanism guarantee that the high type

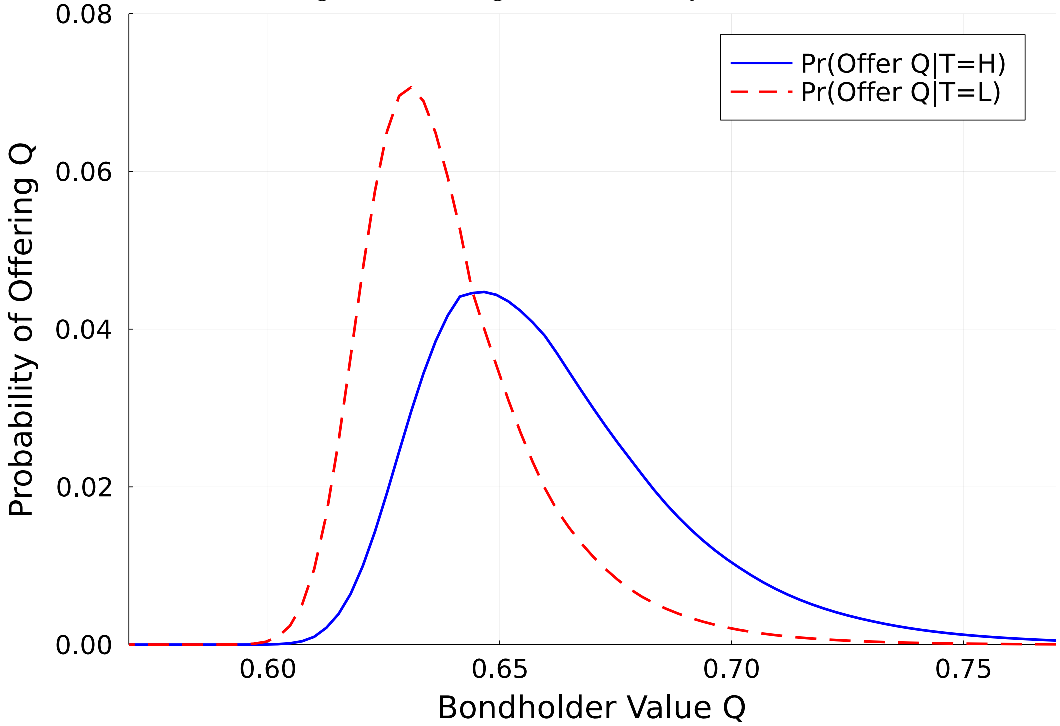
Figure 5: Effects of Signalling Moments



occupies a relatively low debt region of the state space, and therefore rarely defaults. The low type, on the other hand, occupies a higher debt region of the state space and ends up defaulting much more often.

Signalling motives also appear (though not as strongly) in an example set of renegotiation policy functions for the government, which is plotted below in figure 6. In this case, the policy functions of both types are unimodal. However, they display similar patterns to those associated with repayment. The low type makes relatively low offers to lenders most of the time, since it is relatively less worried about lenders rejecting its offer. That said, it does sometimes offer values that the high type plays with nontrivial probability. It does this in order to ensure that lenders accept the offer and to be able to take advantage of the relatively higher prices offered to it in the restructuring phase if it enters with relatively higher reputation. The high type finds default quite painful and evaluates the balance of 1) getting a good deal from lenders, and 2) the probability of exiting default, differently. These considerations lead it to tilt its offers towards higher values in order to ensure that lenders do not reject the deal. In simulations, the high type's offers are accepted 99.7% of the time

Figure 6: Renegotiation Policy Functions



while the low type's offers are accepted 96.7% of the time. When the lender is proposing the deal, the difference is significantly wider. The high type accepts 98.8% of the offers it receives while the low type accepts just 86.9% of the offers it receives.

Before moving on, I want to take a moment to illustrate the long run effects of these signalling motives. Specifically, I want to point out the dramatic effects on the behavior of the high type that they induce. To that end, I re-solve a model with the exact same set of parameters under the assumption that the government's type is public information. I then calculate a few moments of interest, conditional on type. These are listed below in Table 7:

Table 7: Comparison: Differences Across Types

Moment	**Baseline Model**		**Full Info Model**	
	T=H	**T=L**	**T=H**	**T=L**
$\mathbb{E}[B'/Y\|T]$	16.67%	25.85%	21.49%	26.99%
$Pr(T\|d_t = 1)$	2.28%	97.72%	14.51%	85.49%
$\mathbb{E}[\pi\|T]$	0.890	0.147	1	0

The first line of this table shows that moving from the baseline to a full information setting and therefore remove signalling motives causes both types to borrow more in the long run. This is expected, since both types are impatient relative to lenders. However, borrowing by the high type rises almost 5%, while borrowing by the low type rises by only about 1%. This pattern of differences arises because the high type was disciplined much more by the signalling incentives in the asymmetric information setting. When those incentives are removed, it borrows more and defaults much more frequently. As is shown in the second row, the share of defaults performed by the responsible type rises from about 2% to almost 15%, because it now more frequently reaches levels of debt at which default occurs with nontrivial probability. Finally, all of this occurs despite the fact that beliefs are quite accurate in the long run in the asymmetric information setting, as can be seen in the table's third row. But even with high ex ante reputation, the signalling motives are still strong enough that the responsible type still wants to re-prove to lenders that it is indeed responsible.

5.4 Welfare Consequences of Transparency

I now move on to a main policy result of this paper. In this section, I evaluate the welfare effects of policies that disrupt the signalling motives described above. For simplicity, I consider the case in which they are fully eliminated and type is public information forever.[10] This change in the environment is one interpretation of what policies such as transparency initiatives, audit programs, and accountability offices are designed to do. By providing information about why the government is making the policy decisions it makes, it informs the public about the policymaker's type. Transparency therefore takes me from the asymmetric information benchmark all the way to the analogous full information model, eliminating signalling motives forever. First, I evaluate the change in payoffs to the government associated with such a change. Then I evaluate the changes in the welfare of a representative consumer with preferences that potentially differ from the preferences of the government.

In order to provide an exact decomposition of the sources of these variations following Aguiar

[10]These results are robust to weakening this assumption. Instead, I could allow there to be, every period, an possibly imperfectly informative signal about the government's type. This nests the case considered in the text, since the signal can be made perfectly informative.

et al. (2020), I first adjust baseline consumption streams $\hat{c}(.)$ to account for default costs[11]. For example, consider the contribution to the government's value from events occurring in the current period when it enters the period in good standing and decides to default and does not reach a deal with lenders this period:

$$U_0^D(s,T) = (1-\beta_T)(u(y(s)) - \phi_T(s,d_t))$$

In this case I set $c(.)$ such that:

$$(1-\beta_T)u(c(s,T,\epsilon,\pi,b)) = U_0^D(s,T)$$

so consumption is adjusted to account for the effect of the default cost on current period utility. Throughout this section, $c(.)$ will refer to this adjusted consumption value.

Given an initial type $T_0 \in \{H, L, N\}$ (where N indicates the the initial type is randomly drawn from its long run distribution), I define the government payoff or consumer welfare as the expected discounted utility over GDP states and preference shocks when the government starts with zero debt and reputation at the long run probability the government is the high type. For example, the value to the low type under asymmetric information is:

$$\mathbb{E}_{s_0,\epsilon}[V_{AI}(s_0, L, \epsilon, \bar{\pi}, 0)] = \mathbb{E}_0\left[\sum_{t=0}^{+\infty}\left(\Pi_{l=0}^{t-1}\beta_{T(l)}\right)(1-\beta_{T(t)})(u(c_t^{AI}) - \phi_{T(t)}(s_t,d_t))|T(0) = L\right]$$

Since utility is CRRA with relative risk aversion coefficient γ, I can define an overall consumption equivalent change in welfare as ζ in the equation:

$$(1+\zeta)^{1-\gamma}\mathbb{E}_{s_0,\epsilon}\left[V^{AI}(s_0,T_0,\epsilon,\bar{\pi},0)\right] = \mathbb{E}_{s_0,\epsilon}\left[V^{FI}(s_0,T_0,\epsilon,0)\right]$$

Some rearrangement yields:

$$1+\zeta = \left(\frac{\bar{V}_{T_0}^{FI}}{\bar{V}_{T_0}^{AI}}\right)^{\frac{1}{1-\gamma}}$$

[11]Because the relative contribution of the preference shocks to value functions is entirely dependent on the literal number of choices available to the government (and this can be made arbitrarily close to 1 by increasing that number), I consider values net of preference shocks throughout this section.

where $\bar{V}^{AI}_{T_0}$ is the time 0 value under asymmetric information if the initial type is T_0 and $\bar{V}^{FI}_{T_0}$ is its full information counterpart. I also follow Aguiar et al. (2020) by defining a breakdown of this variation into:

1. changes due to different incidence of default costs;
2. changes due to different variability of consumption streams;
3. changes due to different trends in the time path of average. consumption.

To do this, note that I can rewrite $(1+\zeta)$ as:

$$\left(\frac{\bar{V}^{FI}_{T_0}}{\bar{V}^{FI,ND}_{T_0}}\frac{\bar{V}^{AI,ND}_{T_0}}{\bar{V}^{AI}_{T_0}}\right)^{\frac{1}{1-\gamma}} * \left(\frac{\bar{V}^{FI,ND}_{T_0}}{\bar{V}^{FI,NDV}_{T_0}}\frac{\bar{V}^{AI,NDV}_{T_0}}{\bar{V}^{AI,ND}_{T_0}}\right)^{\frac{1}{1-\gamma}} * \left(\frac{\bar{V}^{FI,NDV}_{T_0}}{\bar{V}^{AI,NDV}_{T_0}}\right)^{\frac{1}{1-\gamma}}$$

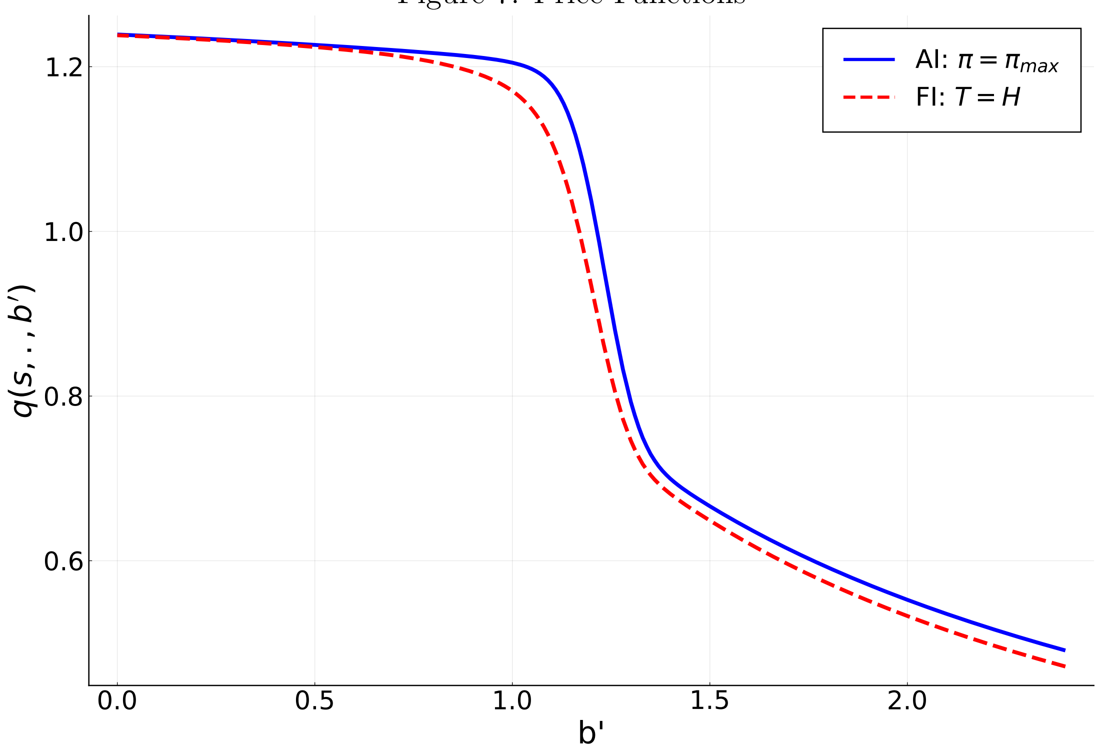
where the value functions with the additional ND superscript are the value functions when default costs are removed and those with the additional NDV superscript are the value functions when default costs are removed and consumption at each time t is set to its mean value across possible paths. The first term is $1+\zeta_D$, the welfare effects of changes in default costs, the second term is $1+\zeta_V$, the welfare effects of changes in the variability of consumption, and the third term is $1+\zeta_T$, the welfare effects of changes in the trend of average consumption over time. Table 8 details the effects on government payoffs of moving from asymmetric information to full information:

Table 8:
Change in Government Payoffs From Transparency

Welfare Change	$T_0 = L$	$T_0 = H$	$T_0 = N$
ζ^G_D	-0.13%	-0.73%	-0.45%
ζ^G_V	-0.05%	-0.10%	-0.08%
ζ^G_T	-0.06%	$+0.34\%$	$+0.15\%$
ζ^G	-0.23%	-0.50%	-0.38%

The bottom line number of Table 8 shows that increased transparency has negative effects on the average payoffs for both the responsible and the irresponsible type. The losses for the responsible type are roughly twice as large as the losses for the irresponsible type. All three channels produce negative effects for the irresponsible type. For the responsible type, however, there is in fact a gain from moving to full information associated with the average time trend of consumption. Since it no longer needs to signal to lenders that it is the responsible type, it can accumulate debt faster and consume more right away. Since it is more impatient than international investors (although not as impatient as the irresponsible type), this is a valuable feature for it. However, it will end up defaulting sooner and more frequently in the full information setting, and those costs end up outweighing the benefits of being able to borrow more in the near future.

Figure 7: Price Functions



Furthermore, the future effects of those signalling motives (or their absence) are constantly reflected in prices. In Figure 7, I plot the price function at the mean level of output under full information, when the government is the high type, and under asymmetric information, when the government is known with certainty to be the high type today. The difference between the two prices is the cumulative disciplining impact of signalling motives throughout the future. While the government ends up borrowing less under asymmetric information due to the signalling motives, the price at which it borrows is consistently higher, which lessens the impact of the lower borrowing levels on auction revenue and therefore consumption.

The equivalent set of values for a representative consumer who is just as patient as the international investors (i.e. has $\beta = \frac{1}{R}$) are:

Welfare Change	$T_0 = L$	$T_0 = H$	$T_0 = N$
ζ_D^C	-0.78%	-1.06%	-0.93%
ζ_V^C	$+0.03\%$	$+0.01\%$	$+0.02\%$
ζ_T^C	$+0.06\%$	$+0.07\%$	$+0.06\%$
ζ^C	-0.69%	-0.98%	-0.84%

Overall, the welfare changes experienced by this representative consumer are dominated by the default cost channel. Since this consumer is significantly more patient than either government and perhaps the most important differences between the two settings is that default happens sooner and more frequently under full information, this should not come as a surprise. This decomposition does illustrate, however, how asymmetric information can shield consumers from the political economy frictions that make their government more impatient than them. The signalling motives induce both government types to act as if they were slightly more patient and had preferences more similar to those of their citizens.

An alternative way to think about these results is to remember that this long bond model, even without types and without endogenous renegotiation, features inefficiently high levels of borrowing (Aguiar and Amador, 2019). There are feasible allocations which are Pareto improving relative to the competitive equilibrium allocation. Since this is a model with incomplete markets, I need to be a little careful and define feasibility more strictly than usual. Specifically, a feasible allocation is one attainable by committing to an infinite sequence of debt issuances while still being subject to the lack of commitment when it comes to the default decision. The signalling motives in the baseline model help push equilibrium borrowing levels down in the long run, moving the competitive equilibrium allocation closer to the allocation a planner would choose when that limited commitment (with respect to the default decision) constraint is imposed. Thus, the signalling motives induced my information asymmetry are acting, on their face, as a substitute for various macroprudential policies intended to limit borrowing, such as fiscal rules.

That said, this result also indicates that literal fiscal rules (limits on debt levels or issuance) may reduce both government payoffs and consumer welfare. According to my results, such rules will disproportionately distort the decisions of the low type (i.e. the type that borrows more will usually be more impacted by limits on borrowing). When the low type's borrowing decision is exogenously constrained, it can become significantly more costly for the high type to prove its identity to lenders (it may have to pull back on borrowing much more than it has to in the benchmark model). If signalling becomes too expensive for the high type, it may simply give up on the whole exercise and start borrowing more. It is therefore plausible that imposing a fiscal rule would actually lead to more borrowing and default!

6 Conclusion

In this paper, I built a flexible model of sovereign borrowing, default, and renegotiation with asymmetric information. I then calibrated this model, disciplining the reputation-related parameters using post-restructuring patterns of interest rate spreads observed in the data. Using the calibrated model, I examined which decisions, in equilibrium, convey the most information. Here, I found that the most important set of decisions in determining a country's reputation are its borrowing decisions. This suggests that the literature's focus on default itself as the key behavior that distinguishes responsible types from irresponsible types may be incomplete, and that more work should be devoted to how differences in borrowing patterns, conditional on repayment, lead to differences in reputation.

After estimating the model and analyzing some notable patterns of behavior present in the calibration, I moved on to validating the model by showing that the predictions it makes about how debt issuance decisions feed into a country's reputation have real quantitative bite. I showed that I can use the model to develop a way to measure countries' reputations using real world data on debt issuance. I then do a few tests of this measure, some stylized and some systematic. In particular, I showed that this model-filtered measure of reputation provides significant additional information in explaining interest rate spreads and default probabilities. This implies that the model is picking up on important patterns in the determination of sovereign borrowers' reputations.

Finally, I used the model to examine the implications of programs which increase transparency in policy making. Here, I showed that by weakening or even removing the signalling motives in place in the benchmark asymmetric information case, such policies can have significant negative effects on both government payoffs and consumer welfare. These effects arise because the presence of signalling motives gives the government a type of pseudo-commitment, a way to prevent its future selves from borrowing too much. Of course, there may exist other channels by which such policies could have positive effects, but it is important to account for this channel in considering whether to implement audit programs when extending debt relief to troubled countries.

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7 Appendix

7.1 Full Definition of Equilibrium

An stationary recursive competitive equilibrium for this environment consists of:

1. Value functions $V, V^R, V_0^D, V^D, V_G^D, V_L^D, \hat{V}_L^D, V_N^D, V^{RS}$;
2. Price functions $q, q^D, q_N^D, \bar{q}_G^D, \hat{q}_G^D, \hat{q}_L^D$;
3. Policy functions $d^\star, b'^\star, Q_G^\star, Q_L^\star, A_G^\star, A_L^\star, b'^\star_{RS}$;
4. Belief update functions $\Gamma^D, \Gamma^R, \Gamma_G^Q, \Gamma_L^A, \Gamma^{RS}$.

which satisfy the following conditions:

1. Default decision optimality: given Γ_D, V_0^D, and V^R, $d^\star$ solves the government's default or repay decision problem and V is the resulting value function.
2. Borrowing decision optimality: given V, Γ^R, and q, $b'^\star$ solves the government's repayment problem and V^R is the resulting value function.
3. Zero profits: given q^D, Γ_D, Γ^R, $d^\star$, and $b'^\star$, q satisfies the functional equation defining prices while in good standing.
4. Offset of initial value of default: given V^D, V_0^D satisfies the equation defining the value of default in the period of default.
5. Government default value if no deal agreed: given V^D, V_N^D is the value function for the government when no deal with lenders is agreed.
6. Lender default value if no deal agreed: given q^D, q_N^D is the defaulted bond price function when no deal is agreed.
7. Ex ante government default value: given V_N^D, V_G^D, and V_L^D, V^D is the value of being in default before resolution of the uncertainty about whether an opportunity to renegotiate arises.
8. Ex ante lender default value: given q_N^D, $\bar{q}_G^D$, and $\hat{q}_G^D$, q^D is the defaulted bond price before

resolution of the uncertainty about whether an opportunity to renegotiate arises.

9. Government deal proposal optimality: given $A_L^\star$, V^{RS}, V_N^D, and Γ_G^Q, $Q_G^\star$ solves the problem of the government when deciding what offer to propose to lenders and V_G^D is the resulting value function.

10. Lender deal acceptance optimality: given q_N^D, $A_L^\star$ solves the lender's problem when deciding whether to accept a deal proposed by the government and $\hat{q}_G^D$ is the resulting price function.

11. Ex ante lender value if receiving proposal: given $\hat{q}_G^D$, Γ_G^Q, and $Q_G^\star$, $\bar{q}_G^D$ is the ex ante price of the bond when an opportunity to renegotiate arises and the government is chosen to be the proposer.

12. Lender deal proposal optimality: $A_G^\star$, Γ_L^A, and q_N^D, $Q_L^\star$ solves the problem of lenders when deciding what offer to propose to the government and $\hat{q}_L^D$ is the resulting price function.

13. Government deal acceptance optimality: V^{RS}, V_N^D, and Γ_L^A, $A_G^\star$ solves the government's problem when deciding whether to accept a deal proposed by lenders and $\hat{V}_L^D$ is the resulting price function.

14. Given $\hat{V}_L^D$ and $Q_L^\star$, V_L^D is the ex ante value to the government when an opportunity to renegotiate arises and lenders are chosen to be the proposer.

15. Government restructuring choice optimality: q, V, and Γ^{RS}, $b_{RS}'^\star$ solves the government's problem of restructuring its debt and V^{RS} is the resulting value function.

16. Bayesian updating: belief updates $\Gamma^D, \Gamma^R, \Gamma_G^Q, \Gamma_L^A, \Gamma^{RS}$ are consistent, respectively, with the policy functions $d^\star, b'^\star, Q_G^\star, A_G^\star, b_{RS}'^\star$ and Bayes' Law.

7.2 Proof of Theorem 1

In this section, I prove that under the assumptions made in the text, an equilibrium must exist. In order to make the exposition of this proof easier to follow, I will adjust some of the notation I have been using. None of these notational adjustments have a material impact on the proof. The most significant of these adjustments is in dealing with belief updates which occur within a period. Instead of using the within period posterior belief as a state variable, I use the beginning of period prior belief as well as the history of actions observed during the current period. Since beliefs are update using Bayes's Law, these two formulations are exactly equivalent.

In order to do that, I must first define (and redefine) some quantities and functions. This section will proceed as follows. First, I define some bounds for value functions and price functions. Second, I define a reduced set of equilibrium objects from which the full set defined above can be recovered. Third, I define an operator which updates that set of equilibrium objects. Fourth, I show that this operator is a continuous mapping of a compact, convex set into itself, and therefore, according to Brouwer's Fixed Point Theorem, must have a fixed point.

Definition 2. *1. Set q^{max} by:*

$$q^{max} = \max\{q^{RF}, \max\{\mathcal{Q}\}\} + \frac{1}{1-\frac{1}{R}}\left(\sigma^R_{\eta,G}log(2) + \sigma^P_{\eta,L}log(N_Q)\right)$$

where q^{RF} is the unique solution to:

$$q^{RF} = \frac{1}{R}(\lambda + \kappa + (1-\lambda)q^{RF})$$

i.e. the risk free price of the bond.

2. Set V^{max} by:

$$u(\max_{s\in\mathscr{S}} y(s)+q_{max}\max_{b'\in\mathscr{B}} b')+\frac{1}{1-\beta_H}\left(\sigma_\epsilon log(N_b+1)+\sigma^P_{\eta,G}log(N_Q)+\sigma^R_{\eta,L}log(2)+\sigma^{RS}_\nu log(N_b)\right)$$

3. Set V^{min} by:

$$\min_{s\in\mathscr{S}}\{u(y(s))\} - \max_{(s,T,d)\in\mathscr{S}\times\{H,L\}\times\{0,1\}}\{\phi_T(s,d)\}$$

The upper bound for q is simply the largest sequence of raw payments lenders can ever receive for it plus the maximum possible contribution of preference shocks to their values.

The upper bound for V is simply the fundamental value (i.e. without preference shocks) of consuming the highest value of consumption possible plus the maximum contribution of preference shocks to the government's value. The lower bound on V is simply the value of being in default in the worst output state with the largest penalty forever. Since always defaulting, always proposing $Q = 0$ and then choosing $b' = 0$, and always declining deals from lenders is a feasible sequence of actions, this provides a lower bound on values for the government.

With these assumptions and definitions in hand, I now define the reduced set of objects which I will use to prove that an equilibrium must exist:

Definition 3. *Let X denote a generic member of the set $\mathscr{X}$, to be defined below:*

$$X = \Big(\bar{V}(s, T, \pi, b), \bar{V}^D(s, T, \pi, b), \bar{q}(s, \pi', b'), \bar{q}^D(s, \pi, b), \bar{\delta}(s, \pi', b'),$$
$$\Gamma^R(b', \pi | s, b), \Gamma^D(\pi | s, b), \Gamma^D_G(d, Q, \pi | s, b), \Gamma^D_L(d, Q, \pi | s, b),$$
$$\Gamma^{RS}_G(d, Q, b', \pi | s, b), \Gamma^{RS}_L(d, Q, b', \pi | s, b) \Big)$$

where:

1. $\bar{V} : \mathscr{S} \times \{H, L\} \times \Pi \times \mathscr{B} \rightarrow [V^{min}, V^{max}]$
2. $\bar{V}^D : \mathscr{S} \times \{H, L\} \times \Pi \times \mathscr{B} \times \rightarrow [V^{min}, V^{max}]$
3. $\bar{q} : \mathscr{S} \times \Pi \times \mathscr{B} \rightarrow [0, q^{max}]$
4. $\bar{q}^D : \mathscr{S} \times \Pi \times \mathscr{B} \rightarrow [0, q^{max}]$
5. $\bar{\delta} : \mathscr{S} \times \Pi \times \mathscr{B} \rightarrow [0, 1]$
6. $\Gamma^R : \mathscr{B} \times \Pi \times \mathscr{S} \times \mathscr{B} \rightarrow [0, 1]$
7. $\Gamma^D : \times \Pi \times \mathscr{S} \times \mathscr{B} \rightarrow [0, 1]$
8. $\Gamma^D_G : \{0, 1\} \times \mathscr{Q} \times \Pi \times \mathscr{S} \times \mathscr{B} \rightarrow [0, 1]$
9. $\Gamma^D_L : \{0, 1\} \times \mathscr{Q} \times \Pi \times \mathscr{S} \times \mathscr{B} \rightarrow [0, 1]$

10. $\Gamma_G^{RS} : \{0,1\} \times \mathscr{Q} \times \mathscr{B} \times \Pi \times \mathscr{S} \times \mathscr{B} \to [0,1]$

11. $\Gamma_L^{RS} : \{0,1\} \times \mathscr{Q} \times \mathscr{B} \times \Pi \times \mathscr{S} \times \mathscr{B} \to [0,1]$

Set $\mathscr{X}$ to be the set of mappings from the product of these 11 domains to the the product of these 11 codomains.

Now, define the feasible sets of choices for the government as:

Definition 4. ***Feasible sets***

1. $\mathscr{F}^R(s,\pi,b) = \{b' \in \mathscr{B} | c(s,\pi,b,b') > 0\}$
2. $\mathscr{F}(s,\pi,b) = \begin{cases} \{0,1\} & \mathscr{F}^R(s,\pi,b) \neq \emptyset \\ \{1\} & \mathscr{F}^R(s,\pi,b) = \emptyset \end{cases}$
3. $\mathscr{F}_G^{RS}(s,\pi,b,d,Q) = \{b' \in \mathscr{B} | c_G^{RS}(s,\pi,b,d,Q,b') > 0\}$
4. $\mathscr{F}_L^{RS}(s,\pi,b,d,Q) = \{b' \in \mathscr{B} | c_L^{RS}(s,\pi,b,d,Q,b') > 0\}$
5. $\mathscr{F}_G^{D}(s,\pi,b,d) = \{Q \in \mathscr{Q} | \mathscr{F}_G^{RS}(s,\pi,b,d,Q) \neq \emptyset\}$
6. $\mathscr{F}_L^{D}(s,\pi,b,d,Q) = \begin{cases} \{0,1\} & \mathscr{F}_L^{RS}(s,\pi,b,d,Q) \neq \emptyset \\ \{0\} & \mathscr{F}_L^{RS}(s,\pi,b,d,Q) = \emptyset \end{cases}$

where the repayment consumption function and auction prices are given by:

$$\begin{aligned} c(s,\pi,b,b') &= y(s) - (\lambda+\kappa)b + q(s,\hat{\pi}',b')(b' - (1-\lambda)b) - i(s,\pi,b,b') \\ q(s,\hat{\pi}',b') &= \hat{\mathbb{E}}[\bar{q}(s,\pi',b')|\hat{\pi}'] \\ \hat{\pi}' &= \Gamma(\Gamma^R(b',\pi|s,b)) \end{aligned}$$

and the restructuring consumption functions and auction prices, for $X \in \{G,L\}$, are given by:

$$\begin{aligned} c_X^{RS}(s,\pi,b,d,Q,b') &= y(s) - Qb + q(s,\hat{\pi}',b')b' - i(s,\pi,b,d,Q,b') \\ q(s,\hat{\pi}',b') &= \hat{\mathbb{E}}[\bar{q}(s,\pi',b')|\hat{\pi}'] \\ \hat{\pi}' &= \Gamma(\Gamma_X^{RS}(d,Q,b',\pi|s,b)) \end{aligned}$$

I can now define the operator T which updates X.

Definition 5. ***The update operator:*** *Set* $T : \mathscr{X} \to \mathscr{X}$ *by:*

$$T(X) = (\bar{V}_{new}, \bar{V}^{D}_{new}, \bar{q}_{new}, \bar{q}^{D}_{new}, \bar{\delta}_{new}, \Gamma^{R}_{new}, \Gamma^{D}_{new}, \Gamma^{D}_{G,new}, \Gamma^{D}_{L,new}, \Gamma^{RS}_{G,new}, \Gamma^{RS}_{L,new})$$

Where the objects in this list are obtained as follows:

First, I obtain the ex ante values of restructuring, choice probabilities associated with restructuring choices, and new restructuring belief update functions. To do this, for each $Q \in \mathscr{F}^{D}_{G}(s, \pi, b, d)$, set $V^{RS}_{G}(s, \pi, b, d, Q)$ by:

$$\begin{aligned} V^{RS}_{G}(s, T, \nu^{RS}, \pi, b, d, Q) = \max_{b' \in \mathscr{F}^{RS}_{G}(s,\pi,b,d,Q)} & (1 - \beta_T)(u(c^{RS}_{G}(s, \pi, b, d, Q, b')) - \phi_T(s, d)) \\ & + \beta_T \hat{\mathbb{E}}[\bar{V}(s', T', \pi', b')|s, T, \hat{\pi}'] + \nu_{RS}(b') \\ \hat{\pi}' = \ & \Gamma(\Gamma^{RS}_{G}(d, Q, b', \pi|s, b)) \end{aligned}$$

Then define the government's ex ante values $\bar{V}^{RS}_{G}(s, T, \pi, b, d, Q)$ and its choice probabilities $p^{RS}_{G}(s, T, \pi, b, d, Q, b')$ by:

$$\begin{aligned} \bar{V}^{RS}_{G}(s, T, \pi, b, d, Q) &= \mathbb{E}[V^{RS}_{G}(s, T, \nu^{RS}, \pi, b, d, Q)] \\ p^{RS}_{G}(s, T, \pi, b, d, Q, b') &= Pr(b'^{RS,\star}_{G}(s, T, \nu^{RS}, \pi, b, d, Q) = b') \end{aligned}$$

Similarly, for each $Q \in \mathscr{Q}$ such that $\mathscr{F}^{D}_{L}(s, \pi, b, d, Q) = \{0, 1\}$ (i.e. accepting the offer Q when that offer is made by lenders results in there being a feasible way to deliver Qb of the consumption good to them), set $V^{RS}_{L}(s, \pi, b, d, Q)$ by:

$$\begin{aligned} V^{RS}_{L}(s, T, \nu^{RS}, \pi, b, d, Q) = \max_{b' \in \mathscr{F}^{RS}_{L}(s,\pi,b,d,Q)} & (1 - \beta_T)u(c^{RS}_{L}(s, \pi, b, d, Q, b')) \\ & + \beta_T \hat{\mathbb{E}}[\bar{V}(s', T', \pi', b')|s, T, \hat{\pi}'] + \nu_{RS}(b') \\ \hat{\pi}' = \ & \Gamma(\Gamma^{RS}_{L}(d, Q, b', \pi|s, b)) \end{aligned}$$

Then define the government's ex ante value $\bar{V}^{RS}_{L}(s, T, \pi, b, d, Q)$ and its choice probabilities

$p_L^{RS}(s, T, \pi, b, d, Q, b')$ by:

$$\bar{V}_L^{RS}(s, T, \pi, b, d, Q) = \mathbb{E}[V_L^{RS}(s, T, \nu^{RS}, \pi, b, d, Q)]$$
$$p_L^{RS}(s, T, \pi, b, d, Q, b') = Pr(b_L'^{RS,\star}(s, T, \nu^{RS}, \pi, b, d, Q) = b')$$

This completes the construction of the set of new objects associated with the restructuring process and that I need in order to update the objects in $T(X)$ associated with the renegotiation and restructuring process, as well as the repayment problem.

I now move on to the updates associated with the renegotiation. I first consider the case where the government has been chosen to propose a deal. First, for each $Q \in \mathscr{F}_G^D(s, \pi, b, d)$, define the value to the government of exiting the period without a deal as:

$$\begin{aligned} V_{G,N}^D(s, T, \pi, b, d, Q) &= (1 - \beta_T)(u(y(s)) - \phi_T(s, d)) \\ &\quad + \beta_T \hat{\mathbb{E}}[\bar{V}^D(s', T', \pi', b)|s, T, \hat{\pi}'] \\ \hat{\pi}' &= \Gamma(\Gamma_G^D(d, Q, \pi|s, b)) \end{aligned}$$

Similarly, define on the same set the value to lenders of exiting a period without a deal as:

$$\begin{aligned} q_{G,N}^D(s, \pi, b, d, Q) &= \frac{1}{R}\hat{\mathbb{E}}[\bar{q}^D(s', \pi', b)|s, \hat{\pi}'] \\ \hat{\pi}' &= \Gamma(\Gamma_G^D(d, Q, \pi|s, b)) \end{aligned}$$

Now define lender lender values and policies after the government has made the feasible offer Q as:

$$\begin{aligned} \hat{q}_G^D(s, \eta_D^R, \pi, b, d, Q) = \max_{A_L \in \{0,1\}} & A_L \Big[Q + \eta^Y\Big] \\ & + (1 - A_L)\Big[q_{G,N}^D(s, \pi, b, d, Q) + \eta^N\Big] \end{aligned}$$

Then define the ex ante lender value and average policies in this case as:

$$q_G^D(s,\pi,b,d,Q) = \mathbb{E}[q_G^D(s,\eta_D^R,\pi,b,d,Q)]$$
$$\bar{A}_L(s,\pi,b,d,Q) = Pr(A_L^\star(s,\eta_D^R,\pi,b,d,Q) = 1)$$

Then I can define the government's problem as:

$$V_G^D(s,T,\eta_D^P,\pi,b,d) = \max_{Q \in \mathscr{F}_G^D(s,\pi,b,d)} \bar{A}_L(s,\pi,b,d,Q)\bar{V}_G^{RS}(s,T,\pi,b,d,Q)$$
$$+ (1-\bar{A}_L(s,\pi,b,d,Q))V_{G,N}^D(s,T,\pi,b,d,Q) + \eta^O(Q)$$

Using this, I can then define, for the case when the government is chosen to be the proposer, the government's ex ante value and choice probabilities:

$$\bar{V}_G^D(s,T,\pi,b,d) = \mathbb{E}[V_G^D(s,T,\eta_D^P,\pi,b,d)]$$
$$p_G^D(s,T,\pi,b,d,Q) = Pr(Q_G^\star(s,T,\eta_D^P,\pi,b,d) = Q)$$

From the lenders' perspective, this implies choice probabilities and ex ante values are:

$$\bar{p}_G^D(s,\pi,b,d,Q) = \Gamma_G^D(d,Q,\pi|s,b)p_G^D(s,H,\pi,b,d,Q) + (1-\Gamma_G^D(d,Q,\pi|s,b))p_G^D(s,L,\pi,b,d,Q)$$
$$\bar{q}_G^D(s,\pi,b,d) = \sum_{Q \in \mathscr{F}_G^D(s,\pi,b,d)} \bar{p}_G^D(s,\pi,b,d,Q)q_G^D(s,\pi,b,d,Q)$$

That completes the construction of required new values and beliefs when the government is chosen to be the proposer. I now move to the case where the lender is chosen to be the proposer. Again, I begin with definitions of values for both parties if they exit the period without a deal. The value to the government in this case is:

$$V_{L,N}^D(s,T,\pi,b,d,Q) = (1-\beta_T)(u(y(s)) - \phi_T(s,d))$$
$$+ \beta_T\hat{\mathbb{E}}[\bar{V}^D(s',T',\pi',b)|s,T,\hat{\pi}']$$
$$\hat{\pi}' = \Gamma(\Gamma_L^D(d,Q,\pi|s,b))$$

Similarly, the value to lenders of exiting a period without a deal is:

$$q^D_{L,N}(s,\pi,b,d,Q) = \frac{1}{R}\hat{\mathbb{E}}[\bar{q}^D(s',\pi',b)|s,\hat{\pi}']$$
$$\hat{\pi}' = \Gamma(\Gamma^D_L(d,Q,\pi|s,b))$$

I now define the government's problem once lenders have made an offer:

$$\begin{aligned}\hat{V}^D_L(s,T,\eta^R_D,\pi,b,d) = \max_{A_G\in\mathscr{F}^D_L(s,\pi,b,d,Q)} & A(\bar{V}^{RS}_L(s,T,\pi,b,d,Q)+\eta^Y) \\ & + (1-A_G)(V^D_{L,N}(s,T,\pi,b,d,Q)+\eta^N)\end{aligned}$$

These values and the associated policy functions let me define the government's ex ante value when lenders make a given offer and its choice probabilities by:

$$V^D_L(s,T,\pi,b,d,Q) = \mathbb{E}[\hat{V}^D_L(s,T,\eta^R_D,\pi,b,d)]$$
$$\bar{A}_G(s,T,\pi,b,d,Q) = Pr(A^\star_G(s,T,\eta^R_D,\pi,b,d)=1)$$

Using these choice probabilities, I can then write, from the lender's perspective, the probability that an offer will be accepted as:

$$\tilde{A}_G(s,\pi,b,d,Q) = \begin{cases} \pi\bar{A}_G(s,H,\pi,b,d,Q)+(1-\pi)\bar{A}_G(s,L,\pi,b,d,Q) & d=0 \\ \Gamma^D(\pi|s,b)\bar{A}_G(s,H,\pi,b,d,Q)+(1-\Gamma^D(\pi|s,b))\bar{A}_G(s,L,\pi,b,d,Q) & d=1 \end{cases}$$

which in turn lets me define the problem of the lender when proposing an offer as:

$$\begin{aligned}q^D_L(s,\eta^P_D,\pi,b,d) = \max_{Q\in\mathscr{Q}} & \tilde{A}_G(s,\pi,b,d,Q)Q \\ & + (1-\tilde{A}_G(s,\pi,b,d,Q))q^D_{L,N}(s,\pi,b,d,Q)+\eta^O(Q)\end{aligned}$$

Finally, I can define lenders' ex ante values when chosen to be the proposer, their choice probabilities, and the government's ex ante values when lenders are chosen to be the proposer:

$$
\begin{aligned}
\bar{q}_L^D(s,\pi,b,d) &= \mathbb{E}[q_L^D(s,\eta_D^P,\pi,b,d)] \\
\bar{p}_L^D(s,\pi,b,d,Q) &= Pr(Q_L^\star(s,\eta_D^P,\pi,b,d) = Q) \\
\bar{V}_L^D(s,T,\pi,b,d) &= \sum_{Q\in\mathcal{Q}} \bar{p}_L^D(s,\pi,b,d,Q)V_L^D(s,T,\pi,b,d,Q)
\end{aligned}
$$

Finally, I can then define values to both sides before it is determined whether a renegotiation will arise in the current period. These are:

$$
\begin{aligned}
V^D(s,T,\pi,b,d) &= \psi(\mu_G\bar{V}_G^D(s,T,\pi,b,d) + (1-\mu_G)V_L^D(s,T,\pi,b,d)) + (1-\psi)V_N^D(s,T,\pi,b,d) \\
q^D(s,\pi,b,d) &= \psi(\mu_G\bar{q}_G^D(s,\pi,b,d) + (1-\mu_G)\bar{q}_L^D(s,\pi,b,d)) + (1-\psi)q_N^D(s,\pi,b,d)
\end{aligned}
$$

where the values if no opportunity arises in this period V_N^D and q_N^D are given by:

$$
\begin{aligned}
V_N^D(s,T,\pi,b,d) &= (1-\beta_T)(u(y(s)) - \phi_T(s,d)) \\
&\quad + \beta_T\hat{\mathbb{E}}[V^D(s',T',\pi',b,0)|s,T,\hat{\pi}'] \\
q_N^D(s,\pi,b,d) &= \frac{1}{R}\hat{\mathbb{E}}[q^D(s',\pi',b)|s,\hat{\pi}'] \\
\hat{\pi}' &= \begin{cases} \Gamma(\Gamma^D(\pi|s,b)) & d=1 \\ \Gamma(\pi) & d=0 \end{cases}
\end{aligned}
$$

Immediately, this allows me to define updated versions of $\bar{V}^D$ and $\bar{q}^D$ as:

$$
\begin{aligned}
\bar{V}_{new}^D(s,T,\pi,b) &= V^D(s,T,\pi,b,0) \\
\bar{q}_{new}^D(s,\pi,b) &= q^D(s,\pi,b,0)
\end{aligned}
$$

I now move to characterize the set of updates associated with values and policies when the government enters the period in good standing. After that I will define the full set of new belief update functions. Given $\bar{V}$, Γ^R, and q, I can define the government's problem if it chooses to repay its debt as:

$$
\begin{aligned}
V^R(s,T,\epsilon,\pi,b) = & \max_{b' \in \mathscr{F}^R(s,\pi,b)} (1-\beta_T)u(c(s,\pi,b,b')) \\
& + \beta_T \hat{\mathbb{E}}[V(s',T',\pi',b')|s,T,\hat{\pi}'] + \epsilon^R(b') \\
\hat{\pi}' = & \Gamma(\Gamma^R(b',\pi|s,b))
\end{aligned}
$$

and, given this and V^D, the government's problem of deciding whether to default is:

$$
V(s,T,\epsilon,\pi,b) = \max_{d \in \mathscr{F}(s,\pi,b)} d(V^D(s,T,\pi,b,d) + \epsilon^D) + (1-d)V^R(s,T,\epsilon,\pi,b)
$$

These let me define update ex ante government values when entering the period in good standing, and the government's choice probabilities:

$$
\begin{aligned}
\bar{V}_{new}(s,T,\pi,b) &= \mathbb{E}[V(s,T,\epsilon,\pi,b)] \\
p_d(s,T,\pi,b) &= Pr(d^\star(s,T,\epsilon,\pi,b) = 1) \\
p_{b'}(s,T,\pi,b) &= Pr(b'^\star(s,T,\epsilon,\pi,b) = b')
\end{aligned}
$$

These type-specific choice probabilities then allow me to define the choice probabilities from the point of view of the lenders, an updated one period ahead default probability, and an updated price function $\bar{q}$:

$$
\begin{aligned}
\bar{p}_d(s,\pi,b) &= \pi p_d(s,H,\pi,b) + (1-\pi)p_d(s,L,\pi,b) \\
\bar{\delta}_{new}(s,\pi',b') &= \mathbb{E}[\bar{p}_d(s',\pi',b')|s] \\
\bar{p}_{b'}(s,\pi,b) &= \pi(1-p_d(s,H,\pi,b))p_{b'}(s,H,\pi,b) + (1-\pi)(1-p_d(s,L,\pi,b))p_{b'}(s,L,\pi,b) \\
\bar{q}_{new}(s,\pi',b') &= \frac{1}{R}\mathbb{E}[\bar{p}_d(s',\pi',b')q^D(s',\pi',b',1) + (1-\bar{p}_d(s',\pi',b'))(\lambda+\kappa) \\
&\quad + \sum_{b'' \in \mathscr{F}^R(s',\pi',b')} \bar{p}_{b''}(s',\pi',b')q(s',\hat{\pi}'',b'')|s] \\
\hat{\pi}'' &= \Gamma(\Gamma^R(b'',\pi'|s',b'))
\end{aligned}
$$

This completes the definitions of new versions of the non-belief update components of X, i.e. $\bar{V}, \bar{V}^D, \bar{q}, \bar{q}^D, \bar{\delta}$. Now, I use the choice probabilities to derive new versions of the belief update functions.

Whenever a choice is not feasible, set the value of the new belief update function to 1 (i.e. certainty that the government is the high type). This assignment rule will be crucial when establishing that the operator T is continuous at points in the space $\mathscr{X}$ where the sets of feasible actions change (i.e. as points where there is at least one consumption value which is exactly 0, so infinitesimal adjustments to the various objects which define consumption may result in that consumption value becoming feasible). The intuition for why this will work proceeds as follows. The high type is the one which places relatively less weight on utility from consumption in the current period. As a choice becomes close to being infeasible, the consumption associated with it gets very close to zero, and the value associated with the choice becomes arbitrarily negative. However, since the high type places less weight on that large negative utility from consumption in the current period, the gap in value for the high type between its optimal choice and the barely feasible grows more slowly than the same difference does for the low type. Therefore, although both choose this barely feasible choice very rarely, the high type chooses it infinitely more often than the low type as the choice becomes infeasible. This intuition will be borne out more rigorously in the main proof of this section. For choices which are feasible, set Γ^R_{new} and Γ^D by:

$$\Gamma^R_{new}(b', \pi|s, b) = \frac{\pi(1 - p_d(s, H, \pi, b))p_{b'}(s, H, \pi, b)}{\bar{p}_{b'}(s, \pi, b)}$$
$$\Gamma^D_{new}(\pi|s, b) = \frac{\pi p_d(s, H, \pi, b)}{\bar{p}_d(s, \pi, b)}$$

Now, set the type specific probabilities of following certain choice paths when in/entering default by:

$$P_G^D(s,T,\pi,b,d,Q) = \begin{cases} p_G^D(s,T,\pi,b,d,Q) & d=0 \\ p_d(s,T,\pi,b)p_G^D(s,T,\pi,b,d,Q) & d=1 \end{cases}$$

$$P_L^D(s,T,\pi,b,d,Q) = \begin{cases} p_L^D(s,T,\pi,b,d,Q) & d=0 \\ p_d(s,T,\pi,b)(1-\bar{A}_G(s,T,\pi,b,d,Q)) & d=1 \end{cases}$$

$$P_G^{RS}(s,T,\pi,b,d,Q) = \begin{cases} p_G^D(s,T,\pi,b,d,Q)p_G^{RS}(s,T,\pi,b,d,Q,b') & d=0 \\ p_d(s,T,\pi,b)p_G^D(s,T,\pi,b,d,Q)p_G^{RS}(s,T,\pi,b,d,Q,b') & d=1 \end{cases}$$

$$P_L^{RS}(s,T,\pi,b,d,Q) = \begin{cases} p_L^D(s,T,\pi,b,d,Q)p_L^{RS}(s,T,\pi,b,d,Q,b') & d=0 \\ p_d(s,T,\pi,b)\bar{A}_G(s,T,\pi,b,d,Q)p_L^{RS}(s,T,\pi,b,d,Q,b') & d=1 \end{cases}$$

Then, at every feasible choice path, I set $\Gamma^D_{G,new}, \Gamma^D_{L,new}, \Gamma^{RS}_{G,new}, \Gamma^{RS}_{L,new}$ by:

$$\Gamma^D_{G,New}(d,Q,\pi|s,b) = \frac{\pi P_G^D(s,H,\pi,b,d,Q)}{\pi P_G^D(s,H,\pi,b,d,Q)+(1-\pi)P_G^D(s,L,\pi,b,d,Q)}$$

$$\Gamma^D_{L,New}(d,Q,\pi|s,b) = \frac{\pi P_L^D(s,H,\pi,b,d,Q)}{\pi P_L^D(s,H,\pi,b,d,Q)+(1-\pi)P_L^D(s,L,\pi,b,d,Q)}$$

$$\Gamma^{RS}_{G,New}(d,Q,b',\pi|s,b) = \frac{\pi P_G^{RS}(s,H,\pi,b,d,Q,b')}{\pi P_G^{RS}(s,H,\pi,b,d,Q,b')+(1-\pi)P_G^{RS}(s,L,\pi,b,d,Q,b')}$$

$$\Gamma^{RS}_{L,New}(d,Q,b',\pi|s,b) = \frac{\pi P_L^{RS}(s,H,\pi,b,d,Q,b')}{\pi P_L^{RS}(s,H,\pi,b,d,Q,b')+(1-\pi)P_L^{RS}(s,L,\pi,b,d,Q,b')}$$

This completes the definition of the operator T. Next, I establish a key property of the consumption functions in this environment, before moving on to the main proof of this section.

Lemma 1. ***Continuity of consumption functions:*** *Let assumptions 4 and 5 hold. Then the values of consumption are continuous in X.*

Proof: Since products of continuous functions and compositions of continuous functions are also continuous, this follows immediately from the assumption that $g(\pi'|\hat{\pi}')$ is continuous in $\hat{\pi}'$ and $\hat{i}(\bar{\delta})$ is continuous.

Before I begin the main proof of this section, I state some definitions for choice probabilities and ex ante values when preference shocks are distributed Generalized Type One Extreme Value or Type One Extreme Value. For Type One Extreme Value shocks, when there is a set of choices $i \in \{1, ..., N\}$ with associated choice values $V_i + e_i$, Dvorkin et al. (2021) show that ex ante values are given by:

$$\bar{V} = E[V_i + e_i] = V_{max} + \sigma log \left(\sum_{j=1}^{N} exp \left(\frac{V_j - V_{max}}{\sigma} \right) \right)$$

and choice probabilities are given by:

$$p_i = \frac{exp\left(\frac{V_i - V_{max}}{\sigma}\right)}{\sum_{j=1}^{N} exp\left(\frac{V_j - V_{max}}{\sigma}\right)}$$

where $V_{max} = \max_{iin\{1,...,N\}} V_i\}$. Note that I have defined these choice probabilities in order to require that denominator be at least 1 (it must be that $V_j - V_{max} = 0$ for at least one element j).

For Generalized Type One Extreme Value shocks, when there is a binary choice $d \in \{0, 1\}$ and a set of choices $i \in \{1, ..., N\}$ which are made when $d = 0$, with associated choice values $V^D + e^D$ and $V_i^R + e_i^R$, respectively, Dvorkin et al. (2021) show that ex ante values are given by:

$$\bar{V} = E[V_i + e_i] = V_{max} + \sigma log \Bigg(exp \left(\frac{V^D - V_{max}}{\sigma} \right) + exp \left(\frac{V_{max}^R - V_{max}}{\sigma} \right) \left(\sum_{j=1}^{N} exp \left(\frac{V_j^R - V_{max}^R}{\sigma \rho} \right) \right)^{\rho} \Bigg)$$

and choice probabilities are given by:

$$p^D = \frac{exp\left(\frac{V^D - V_{max}}{\sigma}\right)}{exp\left(\frac{V^D - V_{max}}{\sigma}\right) + exp\left(\frac{V^R_{max} - V_{max}}{\sigma}\right)\left(\sum_{j=1}^N exp\left(\frac{V^R_j - V^R_{max}}{\sigma\rho}\right)\right)^{\rho}}$$

$$p^R_i = (1 - p^D)\frac{exp\left(\frac{V^R_i - V^R_{max}}{\sigma\rho}\right)}{\sum_{j=1}^N exp\left(\frac{V^R_j - V^R_{max}}{\sigma\rho}\right)}$$

where $V^R_{max} = \max_{i\in\{1,\ldots,N\}} V^R_i$ and $V_{max} = \max\{V^D, V^R_{max}\}$. The fact that Type One Extreme Value shocks (and generalized ones) lead to these types of expressions for ex ante values and choice probabilities will be extremely useful in what follows. Note here that in both cases, ex ante values and choice probabilities are continuous in the choice values V_i (or $V^D, \{V^R_i\}_{i\in\{1,\ldots,N\}}$). Furthermore, note that when I take the limit as one of those values V_k goes to negative infinity, the ex ante values and choice probabilities converge to exactly their values when choice k is removed from the choice set. I now proceed to the proof of the main theorem of this section.

Theorem 2. ***Equilibrium Existence:*** *Suppose that assumptions 1, 2, 3, 4, 5, and 6. Then there exists $X \in \mathscr{X}$ such that X=T(X).*

Proof: Since T maps a compact, convex set into itself, it will be sufficient to establish that T is continuous. First, I show that this is the case for the pieces of $T(X)$ which are not belief updates, i.e. only $\bar{V}_{new}, \bar{V}^D_{new}, \bar{q}_{new}, \bar{q}^D_{new}, \bar{\delta}_{new}$.

As noted above, whenever choice sequences become infeasible, the probability that they are chosen converges to 0 and their influence on the ex ante value of the agent making the choice also converges to 0. Since the consumption functions are continuous in X and the various expected values taken using $\hat{\mathbb{E}}$ are continuous, all the various individual choice values are continuous whenever they are feasible. Furthermore, since the ex ante values and choice probabilities are continuous at points where a choice sequence becomes infeasible, they are then continuous everywhere. Therefore $\bar{V}_{new}, \bar{V}^D_{new}, \bar{q}_{new}, \bar{q}^D_{new}, \bar{\delta}_{new}$ are continuous in X.

Establishing that the belief update functions are also continuous everywhere is a little more difficult. At points in $\mathscr{X}$ where the feasible sets of choices are invariant to small perturbations, the exact same logic invoked above holds. In particular, if there is no choice sequence which has associated consumption function value equal to exactly 0, then I can require that X' be close enough to X that every feasible set always remain the same. This is possible because the consumption values can be uniformly bounded away from zero (either above or below) due to their being only finitely many of them. Given that the feasible set is fixed, there are no choice sequences which, for X, are assigned using Bayes Law but for X' are infeasible and therefore have the posterior belief associated with them update to 1 (the reverse case is also impossible). Since both of these update rules are continuous functions of the consumption whenever consumption is bounded away from 0, T must be continuous at such points in $\mathscr{X}$

The only real difficulty is posed by points in $\mathscr{X}$ where the one of the feasible sets of choices changes. It is not obvious that, at such points, the mapping of old belief update to new belief update will be continuous. These points in $\mathscr{X}$ are those where at least one of the values taken by the consumption functions c, C_G^{RS}, c_L^{RS} is exactly 0. Moving towards such a point along a path with that consumption value strictly greater than 0, the belief update is defined as $\frac{\pi p_{i,H}}{\pi p_{i,H}+(1-\pi)p_{i,L}}$ and both the numerator and the denominator converge to 0. In order to show that the new belief update functions defined by T are continuous in X, I must show that in every case where this can happen, the mapping to the new belief updates is continuous.

Here, there are four possible cases:

1. There is one choice b' which has $c(s, \pi, b, b'|X) = 0$ and $\mathscr{F}^R(s, \pi, b|X) \neq \emptyset$.

2. There is at least one choice b' which has $c(s, \pi, b, b'|X) = 0$ and $\mathscr{F}^R(s, \pi, b|X) = \emptyset$.

3. There is one choice b' which has $c_X^{RS}(s, \pi, b, d, Q, b'|X) = 0$ and $\mathscr{F}_X^{RS}(s, \pi, b, d, Q, b'|X) \neq \emptyset$ for $X \in \{G, L\}$.

4. There is at least one choice b' which has $c_X^{RS}(s, \pi, b, d, Q, b'|X) = 0$ and $\mathscr{F}_X^{RS}(s, \pi, b, d, Q, b'|X) = \emptyset$ for $X \in \{G, L\}$.

The first and third cases are very similar, and the second and fourth cases are very similar. For this reason, I first consider cases 1 and 3 before moving on to cases 2 and 4.

I begin with case 1. Fix (s, π, b) and the name of the choice i which is just infeasible (had $c = 0$) at X_0. And let X' be a point in $\mathscr{X}$ at which choice i is feasible, and suppose that such points X' occur arbitrarily close X_0 (otherwise, continuity holds since the feasible set remains constant in some small enough neighborhood of X_0). In case 1, the choice set for repayment is nonempty at X_0. Therefore, I can uniformly bound away from $-\infty$ the values associated with choices that are feasible at X_0. Let $W > -\infty$ be such a bound. The value of default is bounded by the V^{min} and V^{max}. For any X', let $V_i^R(T)$ be the value to type T when it chooses choice i. The posterior likelihood ratio when i is chosen is given by:

$$\frac{\pi^{post}}{1 - \pi^{post}} = \frac{\pi}{1 - \pi} \frac{p_i^R(H)}{p_i^R(L)}$$

i.e. the posterior likelihood ratio is equal to the prior likelihood ratio multiplied by the action likelihood ratio. In order to prove that the assignment rule for infeasible choice sequences that sets the posterior to 1 yields a continuous mapping from X to the new belief update function, I must show that for X' close enough to X_0, the action likelihood ratio can be made arbitrarily large. The action likelihood ratio can be written as:

$$\frac{p_i^R(H)}{p_i^R(L)} = \frac{1 - p^D(H)}{1 - p^D(L)} * \frac{\sum_{j=1}^N exp\left(\frac{V_j^R(L) - V_{max}^R(L)}{\sigma\rho}\right)}{\sum_{j=1}^N exp\left(\frac{V_j^R(H) - V_{max}^R(H)}{\sigma\rho}\right)} * \frac{exp\left(\frac{V_i^R(H) - V_{max}^R(H)}{\sigma\rho}\right)}{exp\left(\frac{V_i^R(L) - V_{max}^R(L)}{\sigma\rho}\right)}$$

Since V^D and the values of other feasible choices are continuous in X, the ratio of repayment probabilities can be uniformly bounded. Both the numerator and the denominator of the second piece (the summations), are bounded below by 1 and above by the number of possibly feasible choices N_B, so their ratio cannot lie outside $[\frac{1}{N_B}, N_B]$. Therefore, it is sufficient to show that the final piece becomes unbounded as X' gets close to X. Since the exponential is a strictly increasing function, I apply this principle to its argument:

$$\frac{V_i^R(H) - V_i^R(L) - (V_{max}^R(H) - V_{max}^R(L))}{\sigma\rho}$$

Again, since the values of all choices besides i can be uniformly bounded below by W and above by V^{max}, the only term of interest is the difference in values across types at choice i

$(V_i^R(H) - V_i^R(L))$. This can be written as:

$$\frac{V_i^R(H) - V_i^R(L)}{\sigma\rho} = \frac{((1-\beta_H)u(c_i) + \beta_H EV_i(H)) - ((1-\beta_L)u(c_i) + \beta_L EV_i(L))}{\sigma\rho}$$
$$= \frac{(\beta_L - \beta_H)u(c_i) - (\beta_L EV_i(L) - \beta_H EV_i(T))}{\sigma\rho}$$

Since $\bar{V}$ is uniformly bounded, the continuation value terms are uniformly bounded. Since consumption is continuous in X and utility is continuous in consumption and has $\lim_{c\downarrow 0} u(c) = -\infty$, the $u(c_i)$ term can be made to be a negative number of arbitrary magnitude. Since $\beta_L - \beta_H < 0$, this means that their product can be made arbitrarily large. Therefore, the distance between X' and X_0 can be chosen such that the posterior belief at choice i is arbitrarily close to 1. So in case 1, T is continuous. Note that specifying that only a single choice had been just infeasible was not in fact a restriction. The only points at which such a value would have entered this proof are in the default likelihood ratio, and the ratio of the denominators of the repayment choice probabilities, both of which can be uniformly bounded to begin with.

The proof for case 3 (the analogue when a single restructuring choice is just infeasible and the feasible set at X_0 was nonempty) is almost identical (the bounded terms of the action likelihood ratio are different). In that case, when $d = 1$, the action likelihood ratio contains the ratio of default probabilities, the ratio of probabilities that a given Q is offered or accepted, and the ratio of probabilities that b' is chosen. When $d = 0$, it just contains the last two of these ratios. Near X_0, the ratio of default probabilities can be bounded because the default probabilities themselves are both continuous in X. Furthermore, since the ex ante values of restructuring at the Q in question are continuous in X, the ratio of probabilities that this Q is chosen are continuous in X. Therefore that piece can be bounded. This leaves me with just the ratio of probabilities that a given b' value is chosen during the restructuring process:

$$\frac{exp\left(\frac{V_i^{RS}(H) - V_{max}^{RS}(H)}{\sigma^{RS}}\right)}{exp\left(\frac{V_i^{RS}(L) - V_{max}^{RS}(L)}{\sigma^{RS}}\right)} \frac{\sum_{j=1}^{N} exp\left(\frac{V_j^{RS}(L) - V_{max}^{RS}(L)}{\sigma^{RS}}\right)}{\sum_{j=1}^{N} exp\left(\frac{V_j^{RS}(H) - V_{max}^{RS}(H)}{\sigma^{RS}}\right)}$$

As before, the second piece of this ratio (the one involving summations) can be uniformly bounded. Combining the remaining two and then removing the exponential leads to:

$$\frac{V_i^{RS}(H) - V_i^{RS}(L) - (V_{max}^{RS}(H) - V_{max}^{RS}(L))}{\sigma^{RS}}$$

As before, the $V_{max}^{RS}(T)$ terms can be bounded since there are feasible choices at X_0. Substituting for the remaining terms then leads me to a familiar difference:

$$\frac{V_i^{RS}(H) - V_i^{RS}(L)}{\sigma^{RS}} = \frac{(\beta_L - \beta_H)u(c_i) - (\beta_L EV_i(L) - \beta_H EV_i(T))}{\sigma^{RS}}$$

Again, $u(c_i)$ is the only term which is unbounded as X' gets close to X_0 and c_i becomes arbitrarily close to 0. Since it is multiplied by a negative number, this term therefore becomes arbitrarily large as as X' gets close to X_0, meaning that the posterior belief becomes arbitrarily close to 1. Therefore, in case 3, T is continuous.

Now I move to case 2. Fix (s, π, b) and the name of the choice i which is just infeasible (had $c = 0$) at X_0. In this case, at X_0, there were no feasible choices in this state. Set $\hat{V}(T)$ by:

$$\hat{V}^R(T) = \sum_{j=1}^{N} exp\left(\frac{V_j^R(T) - V_{max}^R(T)}{\sigma\rho}\right)$$

When I write the action likelihood ratio now, I need to be more careful with the ratio of repayment probabilities, since at X_0, default occurs with certainty (so the ratio becomes $\frac{0}{0}$). This ratio is:

$$\frac{1 - p^D(H)}{1 - p^D(L)} = \frac{exp\left(\frac{V_{max}^R(H) - V_{max}(H)}{\sigma}\right)\hat{V}^R(H)^\rho}{exp\left(\frac{V_{max}^R(L) - V_{max}(L)}{\sigma}\right)\hat{V}^R(L)^\rho} * \frac{exp\left(\frac{V^D(L) - V_{max}(L)}{\sigma}\right) + exp\left(\frac{V_{max}^R(L) - V_{max}(L)}{\sigma}\right)\hat{V}^R(L)^\rho}{exp\left(\frac{V^D(H) - V_{max}(H)}{\sigma}\right) + exp\left(\frac{V_{max}^R(H) - V_{max}(H)}{\sigma}\right)\hat{V}^R(H)^\rho}$$

Since, in this case, I know that for X' close enough to X_0, the value of default will be the largest of the fundamental choice values for both types, this can be simplified to:

$$\frac{1 - p^D(H)}{1 - p^D(L)} = \frac{exp\left(\frac{V_{max}^R(H) - V^D(H)}{\sigma}\right)}{exp\left(\frac{V_{max}^R(L) - V^D(L)}{\sigma}\right)} * \frac{\hat{V}^R(L)^{-\rho} + exp\left(\frac{V_{max}^R(L) - V_{max}(L)}{\sigma}\right)}{\hat{V}^R(H)^{-\rho} + exp\left(\frac{V_{max}^R(H) - V_{max}(H)}{\sigma}\right)}$$

The second part of this can be uniformly bounded. The first part, however, cannot necessarily be uniformly bounded, because the $V^R_{max}(T)$ will by assumption become arbitrarily large negative numbers as X' gets close to X_0. Combine this with the problematic piece of the action likelihood ratio considered in the previous case to obtain:

$$\frac{exp\left(\frac{V^R_{max}(H)-V^D(H)}{\sigma}\right)}{exp\left(\frac{V^R_{max}(L)-V^D(L)}{\sigma}\right)} * \frac{exp\left(\frac{V^R_i(H)-V^R_{max}(H)}{\sigma\rho}\right)}{exp\left(\frac{V^R_i(L)-V^R_{max}(L)}{\sigma\rho}\right)}$$

or, more compactly:

$$exp\left(\frac{V^D(L)-V^D(H)}{\sigma} + \left(1-\frac{1}{\rho}\right)\frac{V^R_{max}(H)-V^R_{max}(L)}{\sigma} + \frac{V^R_i(H)-V^R_i(L)}{\sigma\rho}\right)$$

The V^D terms, as well as the continuation value terms inside the V^R's are uniformly bounded and therefore will not play any further role. Removing the exponential and multiplying through by $\sigma * \rho$, the piece of interest is then:

$$\begin{aligned}
&(\rho-1)\Bigg((1-\beta_H)u(c^\star(H)) - (1-\beta_L)u(c^\star(L))\Bigg) + (\beta_L-\beta_H)u(c_i) \\
&= (\rho-1)\Bigg((1-\beta_H)\Big(u(c^\star(H)) - u(c^\star(L))\Big) - (\beta_H-\beta_L)u(c^\star(L))\Bigg) + (\beta_L-\beta_H)u(c_i) \\
&= (\rho-1)(1-\beta_H)\Big(u(c^\star(H)) - u(c^\star(L))\Big) + (\beta_L-\beta_H)\Big(u(c_i) - u(c^\star(L))\Big) + \rho(\beta_L-\beta_H)u(c^\star(L))
\end{aligned}$$

To complete the proof, I now need merely show that the difference in utility terms are bounded. Although it is not necessary, I will provide uniform bounds for both. Suppose that $c^\star(T)$ is the highest value choice for type T. Then for any alternate choice yielding consumption c^{alt} and continuation value $EV^{alt}(T)$:

$$\begin{aligned}
(1-\beta_T)u(c^\star(T)) + \beta_T EV^\star(T) &\geq (1-\beta_T)u(c^{alt}(T)) + \beta_T EV^{alt}(T) \\
\frac{\beta_T}{1-\beta_T}(EV^\star(T) - EV^{alt}(T)) &\geq u(c^{alt}(T)) - u(c^\star(T))
\end{aligned}$$

Since the expected value terms are uniformly bounded, this provides an explicit upper bound on difference between flow utilities from two choices when one of those two choices is the highest value choice for at least one of the types. Thus the term:

$$(\rho - 1)(1 - \beta_H)\Big(u(c^\star(H)) - u(c^\star(L))\Big)$$

can be uniformly bounded both above and below. Furthermore, since $u(c_i) - u(c^\star(L))$ can be bounded above and $(\beta_L - \beta_H) < 0$, the term:

$$(\beta_L - \beta_H)\Big(u(c_i) - u(c^\star(L))\Big)$$

can be uniformly bounded below. The only remaining term is then:

$$\rho(\beta_L - \beta_H)u(c^\star(L))$$

As was the case before, this term becomes unboundedly large as X' gets close to X_0 and the largest possible consumption value converges to 0. Therefore, the distance between X' and X_0 can be chosen such that the posterior belief at choice i is arbitrarily close to 1. So in case 3, T is continuous. The proof for case 4 is almost identical.

In case 4, the action likelihood ratio contains some different pieces. If $d = 1$, it contains the ratio of default probabilities. Since the default probabilities are continuous, this term can be bounded. Regardless of the value of d, the action likelihood ratio contains the probability of the government offering (or accepting) choice Q, and the probability that it then chooses b' during the restructuring process. These are the probabilities of interest. Labeling the Q choice which is just infeasible at X_0 i and one of the just infeasible b' choices k, the likelihood ratio for choosing Q_i is:

$$\frac{exp\left(\frac{\bar{V}_i^{RN}(H) - \bar{V}_{max}^{RN}(H)}{\sigma^{RN}}\right)}{exp\left(\frac{\bar{V}_i^{RN}(L) - \bar{V}_{max}^{RN}(L)}{\sigma^{RN}}\right)} \frac{\sum_{j=1}^N exp\left(\frac{\bar{V}_j^{RN}(H) - \bar{V}_{max}^{RN}(L)}{\sigma^{RN}}\right)}{\sum_{j=1}^N exp\left(\frac{\bar{V}_j^{RN}(H) - \bar{V}_{max}^{RN}(H)}{\sigma^{RN}}\right)}$$

where $\bar{V}_i^{RN}(T)$ is the ex ante value of proposing or accepting Q_i, and the ex ante value of the highest deal is $\bar{V}_{max}^{RN}(T)$. The second ratio (the summations) can be uniformly bounded, as usual. Since proposing $Q = 0$ and rejecting an offer is always a feasible choice, the

$\bar{V}^{RN}_{max}(T)$ terms are uniformly bounded. So the only term of concern here are the two $\bar{V}_i^{RN}(T)$ values.

If the government is accepting the deal, these are simply this ex ante values of proceeding to the restructuring phase having agreed to deliver Q to every bondholder. If the government is proposing the deal, then these are the ex ante restructuring values multiplied by the probability lenders accept the deal plus the values of remaining in default multiplied by the probability that lenders reject the deal, so in both cases, they can be writtin as:

$$\bar{V}_i^{RN}(T) = \alpha \bar{V}_i^{RS}(T) + (1-\alpha) V_i^D(T)$$

with $\alpha \in [0,1]$ and $V_i^D(T)$ denoting the value of remaining in default after the government makes its choice and the ex ante value of entering the restructuring process is:

$$\bar{V}_i^{RS}(T) = V_{i,max}^{RS}(T) + \sigma^{RS} log\left(\sum_{l=1}^{M} exp\left(\frac{V_{i,l}^{RS} - V_{i,max}^{RS}}{\sigma^{RS}}\right)\right)$$

As before, the second term can be uniformly bounded. Then the only piece of the renegotiation phase action likelihood ratio which can potentially be unbounded can be written as:

$$exp\left(\alpha \frac{V_{i,max}^{RS}(H) - V_{i,max}^{RS}(L)}{\sigma^{RN}}\right)$$

The action likelihood ratio associated with the restructuring process itself is:

$$\frac{exp\left(\frac{V_{i,k}^{RS}(H) - V_{i,max}^{RS}(H)}{\sigma^{RS}}\right)}{exp\left(\frac{V_{i,k}^{RS}(L) - V_{i,max}^{RS}(L)}{\sigma^{RS}}\right)} \frac{\sum_{l=1}^{M} exp\left(\frac{V_{i,l}^{RS}(L) - V_{i,max}^{RS}(L)}{\sigma^{RS}}\right)}{\sum_{l=1}^{M} exp\left(\frac{V_{i,l}^{RS}(H) - V_{i,max}^{RS}(H)}{\sigma^{RS}}\right)}$$

As has been true throughout this proof, the piece involving summations can be uniformly bounded. This leaves me with:

$$exp\left(\frac{(V_{i,k}^{RS}(H) - V_{i,k}^{RS}(L)) - (V_{i,max}^{RS}(H) - V_{i,max}^{RS}(L))}{\sigma^{RS}}\right)$$

Combining this with the piece from the renegotiation process, removing the exponential, and multiplying through by σ^{RS} then yields:

$$\Big(V_{i,k}^{RS}(H) - V_{i,k}^{RS}(L)\Big) - \Big(1 - \frac{\alpha\sigma^{RS}}{\sigma^{RN}}\Big)\Big(V_{i,max}^{RS}(H) - V_{i,max}^{RS}(L)\Big)$$

I then substitute using the definitions of each of these values (and drop the continuation value terms, since they, again, can be uniformly bounded) to obtain:

$$\Big((1-\beta_H)u(c_{i,k}) - (1-\beta_L)u(c_{i,k})\Big) - \Big(1 - \frac{\alpha\sigma^{RS}}{\sigma^{RN}}\Big)\Big((1-\beta_H)u(c_i^\star(H)) - (1-\beta_L)u(c_i^\star(L))\Big)$$

Similarly to how things worked in case 2, rearranging terms yields:

$$\begin{aligned}&\Big(\beta_L - \beta_H\Big)\Big(u(c_{i,k}) - u(c_i^\star(L)\Big) - \Big(1 - \frac{\alpha\sigma^{RS}}{\sigma^{RN}}\Big)(1-\beta_H)\Big(u(c_i^\star(H)) - u(c_i^\star(L))\Big)\\ &+ \frac{\alpha\sigma^{RS}}{\sigma^{RN}}\Big(\beta_L - \beta_H\Big)u(c_i^\star(L))\end{aligned}$$

Since differences of the form $u(c_i^{alt}(T)) - u(c_i^\star(T))$ can be uniformly bounded above in the exact same way they were in case 2, the only term here which can potentially be unbounded is:

$$\frac{\alpha\sigma^{RS}}{\sigma^{RN}}\Big(\beta_L - \beta_H\Big)u(c_i^\star(L))$$

As X' gets arbitrarily close to X_0, all feasible consumption values become arbitrarily close to 0, so the utility from consumption becomes an arbitrarily large negative number. Since it is multiplied by another negative number $\beta_L - \beta_H < 0$, this term becomes arbitrarily large. Therefore, as X' becomes arbitrarily close to X_0, the new posterior belief becomes arbitrarily close to 1, which is exactly its value at X_0. Therefore, in case 4, T is continuous. Note that by proving case 4 and showing that the mapping to $\Gamma_{X,new}^{RS}$ is always continuous, I also show that the mapping to $\Gamma_{G,new}^{RN}$ (used when the government proposes Q but lenders decline the offer) is continuous when some Q is just infeasible at X_0. This follows because the individual action likelihood ratios associated with every choice the government could make after proposing Q all become infinitely large as X' gets close to X_0. Therefore the ratio of the sums of those likelihoods must also become infinitely large, so the posterior belief which occurs just upon seeing Q also converges to 1 as X' becomes arbitarily close to X_0.

Since I have now shown that in all four potentially problematic cases, T is continuous, I have now established that T is a continuous operator mapping a compact, convex subset of a finite dimensional Euclidean Space into itself. Therefore, by Brouwer's Fixed Point Theorem, the operator T has a fixed point, so an equilibrium must exist. This completes the proof.

The definition of T shows how to recover the full set of equilibrium objects described in the main text from a given X. Therefore, the existence of a fixed point of T is equivalent to the existence of an equilibrium, proving Theorem 1.

7.3 Details of Computational Algorithm

The reduced set of objects described in the section proving the existence of an equilibrium, in definition 3, is similar to the set of objects used to solve the model numerically and assess convergence. The set used for the computation is:

1. The continuation value functions $Z(s, T, \pi', b')$ and $Z^D(s, T, \pi', b')$, given by:

$$
\begin{aligned}
Z(s, T, \pi', b') &= \mathbb{E}[V(s', T', \epsilon', \pi', b')|s, T] \\
Z^D(s, T, \pi', b') &= \mathbb{E}[V^D(s', T', \pi', b')|s, T]
\end{aligned}
$$

2. The price functions $q(s, \pi', b')$ and $q_N^D(s, \pi, b)$ and the expected probability of default $\delta(s, \pi', b')$.
3. The belief update functions $\Gamma^R(b', \pi|s, b)$, $\Gamma^D(\pi|s, b)$, $\Gamma_G^Q(Q, \pi|s, b)$, $\Gamma_L^A(Q, \pi|s, b)$, and $\Gamma^{RS}(b', \pi|s, W)$.

In short, we have the continuation value functions, the price functions, the expected probability of default, and the belief update functions. These include all the main forward looking pieces of the model (there are other price and value functions, of course, but they can be derived based only on the above set of objects and within-period optimization). Therefore, we use the above set as the list to assess convergence.

The above objects are defined on grids of their arguments. Therefore, in addition to the grid for $T \in \{L, H\}$, I must define the grids for $s \in \mathscr{S}$, $b \in \mathscr{B}$ (and $b' \in \mathscr{B}'$, possibly identical to $\mathscr{B}$, but also possibly different, in order to allow the government to choose from a finer set of debt values), $Q \in \mathscr{Q}$, $W \in \mathscr{W}$ and $\pi \in \Pi$. For s, which defines the grid for GDP values $y(s)$, I use 51 points evenly spaced in logs spread across a space spanning six of the logged variable's long run standard deviations and centered at its mean (i.e. the interval $[\mathbb{E}[log(y(s))] - 3\sigma[log(y(s))], \mathbb{E}[log(y(s))] + 3\sigma[log(y(s))]]$). For b, I use 241 evenly spaced points on $[0, 2.4]$[12]. For b', I use 601 evenly spaced points on $[0, 2.4]$. For Q, I use 501 evenly spaced points on $[0, q_{rf}]$, where q_{rf} is the risk free price of debt. For W, I use 241 evenly spaced points on $[0, 2.4 * q_{rf}]$. For Π, I used 41 points equally spaced in log-odds ratio space across $[1 - p_{LL}, p_{HH}]$.[13] I use the log odds ratio, rather than the raw probability, because it tends to perform significantly better in terms of speed of convergence without substantially affecting the model's predictions.

Given a current guess for the set of objects listed above, a single iteration proceeds as follows in order to generate a new guess:

1. Given the baseline set of objects, solve the government's restructuring problem. Using the policy functions, generate a new guess for $\Gamma^{RS}(b', \pi | s, W)$.

2. Using the baseline set of objects as well as the new solution to the government's restructuring problem, solve the problem of government and the lenders when each is the receiver of a proposal. Using the government's policy function, generate a new guess for $\Gamma_L^A(Q, \pi | s, b)$.

3. Using the baseline set of objects as well as the new solution to the government's restructuring problem and each party's renegotiation problem when receiving a proposal, solve the renegotiation problem of the government and the lenders when each is proposing a deal. Using the government's policy function, generate a new guess for $\Gamma_G^Q(Q, \pi | s, b)$.

4. Using the baseline set of objects as well as the solutions derived so far, generate new guesses of $Z^D(s, T, \pi', b')$ and $q_N^D(s, \pi, b)$.

5. Using the baseline set of objects and those derived in the prior steps, solve the govern-

[12] Because the model is quarterly, this corresponds to $0 - 60\%$ of long run average annual GDP.

[13] For the renegotiation and restructuring steps, this needs to be expanded to cover the full space [0,1]. For those, I add the endpoints 0 and 1 then put 39 points in $[1 - p_{LL}, p_{HH}]$.

ment's problem when it enters a period in good standing. Using the solution, generate new guesses of $Z(s, T, \pi', b')$, $q(s, \pi', b')$, $\delta(s, \pi', b')$, $\Gamma^D(\pi|s, b)$, and $\Gamma^R(b', \pi|s, b)$.

6. Check the sup-norm distance between all objects. If it is less than 10^{-5}, stop. Otherwise, update guesses using rules of the form

$$f_{next}(.) = \zeta_F f_{old}(.) + (1 - \zeta_F) f_{new}(.)$$

where $F \in \{V, q, \Gamma\}$ and return to step 1. This type of rule updates the old guess by moving fraction $(1 - \zeta_F)$ of the distance towards the new guess. The ζ_F may be specific to the type of function involved (value, price, or belief). In general, to ensure convergence, updates of the the belief functions tend to require more smoothing than those of the price functions, which in turn tend to require more smoothing than those of the value functions.