Discriminatory Price Auctions and Self-Fulfilling Crises *

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Abstract

In this paper, we study how different auction protocols make the government more or less vulnerable to multiplicity driven by self-fulfilling prophecies. First, we describe how using a discriminatory price protocol may create a new type of static multiplicity. Because investors pay as bid, equilibrium bids depend on investors' beliefs about how much debt the government is going to issue in a given auction, and different beliefs may support different equilibria. We then show that for linear utility, the equilibrium under a discriminatory price protocol is unique. We conjecture that this static multiplicity requires a substantial level of risk aversion (as in Stangebye (2020)) and for low risk aversion, we should still expect uniqueness. Finally, we show that using the discriminatory price protocol eliminates the type of multiplicity found in Calvo (1988). With this, we provide a rational for the use of discriminatory price auctions, particularly under low risk aversion, where the equilibrium is unique.

JEL Codes: D44, E43, F34, F41, G15, H63.

Keywords: Sovereign debt auctions, default risk, multiplicity.

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1 Introduction

Both policymakers and academics have for decades been intrigued by the possibility of equilibrium multiplicity in sovereign debt markets (see for example Calvo (1988) or Cole and Kehoe (2000)). In particular, there has often been concern about the possibility of both a "bad equilibrium" and a "good equilibrium," and the extent to which policies can be designed to select the good one. In this paper, we study the relationship between the potential for multiplicity and the protocols used in the primary market for sovereign bonds (although our results also apply to settings involving other kinds of borrowers). Most government debt is issued in auctions. In these auctions, investors submit bids consisting of the highest price they are willing to pay to purchase a unit of debt, and how much they are willing to buy. Then, the government chooses which bids to accept. There is wide variation across countries in auction protocols, i.e. the set of rules determining how much each winning bid pays. OECD (2023) found 40 of 41 countries surveyed used auctions. Of those, 12 used uniform price auctions, 15 used discriminatory price auctions and 13 used both.

In this paper, we use an analytical model to explore the role of how debt is issued in mitigating or exacerbating certain key frictions in sovereign debt markets. In Alves Monteiro and Fourakis (2023b), we performed a thorough analysis of the impact of the discriminatory price protocol in exacerbating limited commitment frictions and dilution incentives. In this paper, we focus on how different auction protocols make the government more or less vulnerable to multiplicity driven by self-fulfilling expectations.

We first present a new type of "static multiplicity" arising from the choice of auction protocol. By static multiplicity, we mean the possibility of multiple equilibria in the current period, taken as given a single set of future payoffs.[1] This new source of multiplicity does not require modifying the timing of decisions within a period (as in Cole and Kehoe (2000)), nor does it require the use of long-term debt (as in Lorenzoni and Werning (2019)).

[1] This contrasts with certain other kinds of multiplicity in the literature on sovereign default, which often rely on different expectations of payoffs inducing different equilibrium strategies in the current period. When these equilibrium strategies are consistent with the expectations that produce them, the full game has multiple equilibria. See Aguiar and Amador (2020) for an example of this type of "dynamic multiplicity."

In fact, this multiplicity arises in the simple two period environment under discriminatory price auctions where investors “pay-as-bid”. The mechanism is reminiscent of the dynamic multiplicity introduced with long-term debt, as it relies on the non-exclusivity of debt and the risk of dilution. That is, as the government accepts more bids within an auction, the probability of default goes up and the value of debt goes down, i.e. the value of the asset is diluted. The main difference between our “static multiplicity” and the multiplicity inherent in environments with long-term debt is that ours is not dynamic. It does not depend on beliefs about future fiscal policy, but rather on beliefs about fiscal policy in different states of the world, within an period.

When debt is sold via a uniform price protocol, the bid for any increment that the government might accept is pinned down uniquely by future payoffs. On the other hand, when investors “pay-as-bid,” bids for each increment ever accepted depend on the entire distribution of government debt issuance decisions. This is because competitiveness and investor optimality require that such bids equal the expected value of the bond purchased, conditional on the bid being executed. The value of a bond is pinned down by total government borrowing in the auction. For almost all bids, there will be multiple possible total debt issuance outcomes when that bid is accepted. The ex-ante expected value of bidding on each increment of debt issuance therefore depends on investors’ beliefs about how much the government will borrow in the auction. Different beliefs can support different equilibria. We show that the potential for this kind of static multiplicity hinges on the curvature of flow utility. For linear utility (and arbitrarily small amounts of noise in the government’s decision problem), we prove that the equilibrium is unique, and we conjecture that it remains unique for low enough values of risk aversion.

We then introduce the alternative auction protocols, as in Alves Monteiro and Fourakis (2023b), into a model similar to that in Ayres et al. (2023) to evaluate the interaction of Calvo (1988) type multiplicity with discriminatory price auctions. The key difference between the environment of Eaton and Gersovitz (1981), the starting point of the vast majority of the modern literature on sovereign default, and the setting of Calvo (1988) is the definition of the government’s choice variables when making fiscal policy. In Eaton and Gersovitz (1981) and most papers on sovereign default since then, the government chooses the amount of debt to be issued, whereas in Calvo (1988), the government chooses amount of revenue to be collected (or, equivalently, the budget deficit). When the government issues debt using a uniform price protocol, assuming that deficits are the choice variable may result in a static multiplicity (this is a focus of both Calvo (1988) and Ayres et al. (2023)). On the other hand, assuming that debt issuances are the choice variable leads to static uniqueness. When the government issues debt using a discriminatory price protocol, we show that the set of equilibria is independent of assumptions about the nature of the choice variable.

By showing that the choice of auction protocol has implications for the government's vulnerability to self-fulfilling crises, this paper shows that multiplicity can be found in the canonical setting of Eaton and Gersovitz (1981) when alternative auction protocols are considered. At the same time, this paper contributes to the literature that studies differences in these two types of auctions, highlighting how the choice of protocol affects the government's vulnerability to certain kinds of self-fulfilling runs. In particular, discriminatory price auctions guarantee static uniqueness of equilibrium in situations where uniform price protocols are vulnerable to static multiplicity.

2 Literature Review

This paper builds the literature that studies equilibrium uniqueness and multiplicity in models of borrowing with strategic default. One key condition for uniqueness in the classic setting of Eaton and Gersovitz (1981) is an assumption about the timing of decisions. First the government decides whether to default or not. After that, if it repays, it decides how much to borrow. One canonical example of multiplicity is Cole and Kehoe (2000), which reverses this timing assumption in order to allow for self-fulfilling panics. In Cole and Kehoe (2000), first, the government borrows and only then decides whether or not to default. As such, for part of the state space, a failed auction triggered by expectations of future default leads to default, which could have been prevented if the auction had been successful.[2] We show that, even under Eaton and Gersovitz (1981) timing assumptions, modeling the discriminatory price auction opens the door to expectation driven equilibrium multiplicity.

In Calvo (1988), the key fiscal policy variable is the government deficit (rather than the amount of debt to be issued, as in Eaton and Gersovitz (1981)), and there may be multiple equilibrium interest rates, each associated with a different level of borrowing. Specifically, there can be one equilibrium with a high interest rate and therefore high debt accumulation that generates high default probabilities that justify the high interest rates, as well as another with low interest rates and therefore low debt accumulation that generates low default probabilities that justify the low interest rates. Ayres et al. (2018) argues that this mechanism found in Calvo (1988) is of interest when the fundamental uncertainty is bimodal, with both good and bad times. Ayres et al. (2023) builds on Ayres et al. (2018) by showing that this multiplicity is also quantitatively relevant. The authors incorporate an endowment growth rate process with persistent high and low growth regimes into a standard sovereign debt model and show that the model features self-fulfilling debt crises when calibrated to the data on both Emerging Market Economies and Advanced Economies. We show that this particular type of multiplicity is peculiar to the uniform price protocol. When the discriminatory price protocol is used, we show that assumptions about the nature of the choice variable (whether it is debt issuance or the deficit) have no effect on the number of equilibria.

At a dynamic level, Aguiar and Amador (2020) and Lorenzoni and Werning (2019) show that the use of long-term debt, as in Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), opens the door to multiplicity, even when considering the timing under Eaton and Gersovitz (1981). This dynamic multiplicity arises from investors' self-fulfilling beliefs regarding future borrowing. That is, the government either pursues a fiscal policy that reduces debt or one of high debt and eventual default. Both fiscal policies can be an equilibrium under the right beliefs and consistent prices. The type of multiplicity we find is similar to this dynamic multiplicity in spirit but is static in nature as it depends

[2] Bocola and Dovis (2019) and Bianchi and Mondragon (2022) build on Cole and Kehoe (2000) type runs. Broner et al. (2014) and Galli (2021) explore multiplicity in an economy with capital investment.

on beliefs about fiscal policy in different states of the world, within an auction in a given period.

Related papers focusing on limiting multiplicity (or proving actual uniqueness) are Auclert and Rognlie (2016) and Aguiar and Amador (2019) and Bolivar (2023). Both Auclert and Rognlie (2016) and Aguiar and Amador (2019) prove that Markov perfect equilibrium is unique in classic setting of Eaton and Gersovitz (1981). Whereas they focus on ruling out dynamic multiplicity, we focus on limiting the potential for the type of static multiplicity described above. We show that when the government is risk neutral and uses a discriminatory price protocol, static multiplicity is impossible. We conjecture that this static uniqueness also holds for low levels of risk aversion.

This paper also builds on the literature comparing different auction protocols used to issue sovereign debt. Related papers here include Cole et al. (2018), Pycia and Woodward (2023), Cole et al. (2022) and builds directly on Alves Monteiro and Fourakis (2023b). The first three papers consider a static auction model with asymmetric information across bidders and exogenous asset quality. Alves Monteiro and Fourakis (2023b) focused on incorporating different auction protocols into an infinite horizon, dynamic model of government borrowing and default. It showed that strategic interaction between a government with discretion on the quantity sold and optimizing investors matter. In particular, investors know that distinct protocols induce different debt issuances by the government. The analysis abstracted from self-fulfilling equilibria, comparing instead the best equilibrium under each auction protocol. This paper shows that self-fulfilling crises may arise even in a simplified two period model with a single discriminatory price auction.

3 A General Model of Debt Auctions

Here we present an environment close to the one in Alves Monteiro and Fourakis (2023b), with slight changes regarding debt contracts and prices, detailed below. Time is discrete and infinite, $t = \{0, 1, 2, \dots\}$. The small open economy is populated by a government that borrows from a unit continuum of identical, competitive, risk neutral and deep pocketed foreign investors. These investors' discount factor is given by δ.

The government is benevolent and maximizes the welfare of the small open economy, endowed with y in each period. Preferences over streams of consumption are as follows:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t u(c_t)\right]$$

where u is strictly increasing and concave and $\beta \in (0,1)$ denotes the discount factor.

There is a public exogenous state of the world $s \in \mathcal{S}$, which follows a Markov process that governs the endowment $y(s)$ and expected public spending. The private exogenous state of the world includes $x \in \mathcal{X}$, and a vector of preference shocks for the government, all i.i.d over time. Government spending, $g(s,x)$ depends on the public expectation included in s, as well as on the budget surprise privately observed by the government and indexed by x.

Debt Contract. The government borrows using a defaultable long term bond. We assume that there is a finite set $\mathcal{B}$ of values that the government's debt level b can take. We follow Hatchondo and Martinez (2009), and model debt as a contract promising a stream of exponentially declining coupon payments. Specifically, a new debt issuance of ℓ_t units of debt at time t, promises to pay the following stream of payments starting at period $t+1$:

$$\ell_t, \quad (1-\lambda)\ell_t, \quad (1-\lambda)^2\ell_t, \quad \ldots$$

This debt contract formulation is convenient because it allows to write debt payments B_t recursively as

$$b_t = (1-\lambda)b_{t-1} + \ell_{t-1}$$

Auctions. To issue ℓ_t the government runs an auction and investors submit bid schedules. We consider the two types of protocols typically used for auctioning sovereign debt: the uniform price protocol (UP) and the discriminatory price protocol (DP). The protocol determines which bids are accepted by the government and at which prices they are executed.

Investors submit bid functions, a tuple $(p, b, K) = \left(\{p_k, b_k\}_{k\in\{1,...,K\}}\right)$ with $K < \infty$. A bid is a pair (p_k, b_k), representing the highest price p_k an investor is willing to pay to purchase b_k units of debt. The government sorts bids from highest to lowest (price) and accepts all bids until it is able to borrow ℓ units. The lowest accepted price, the one that clears the auction, is referred to as the marginal price of the auction, P_c.

Under UP, all accepted bids are executed at the same price, which corresponds to the marginal price of the auction, P_c. We analyse the most common DP, the "*pay-as-bid*", under which accepted bids are executed at the respective bid price. The auction protocol is known by all agents before the auction.

Default. If the government chooses to default, the country is excluded from financial markets and suffers a flow utility cost of $h(s)$. The country regains access to financial markets with probability η. Reentry is done through restructuring. Upon reentry the government is liable for a fraction $(1 - \tau)$ of the debt, B, it had prior to the default event.

Timing. As a baseline we consider the setting introduced in Eaton and Gersovitz (1981). The timing of events within a period is as follows.

1. The exogenous state variables are realized at the beginning of the period.
2. If in good standing, the government chooses whether to default.

3.1. If the government entered the period in good standing and chose to repay $(d = 0)$:

   (a) The government runs an auction;

   (b) Investors submit bid functions after observing the public state s;

   (c) The government chooses B' and P_c, given the aggregate bid function.

3.2. If the government chose to default $(d = 1)$ or entered the period in bad standing, it is excluded from financial markets and cannot borrow.

   (a) Next period, with probability η the government regains access to financial markets, and with probability $(1 - \eta)$ remains excluded.

In this setting, the government effectively chooses the units of debt to be issued by choosing a point in the aggregate bid function, acting as a monopolist.

Government. Let there be some continuation value for the government[3] $V(s', b')$ and some continuation value for the lender $Q(s', b')$. In particular, let $Q(\cdot)$ denote the beginning of period value for the lenders. The state x is i.i.d. over time and therefore does not show up in the continuation value functions.

Consider the following problem

$$V^R(s,x,b) = \max_{c,\ell} \left\{ u(c) + \beta \mathbb{E}[V(s', \mathcal{B}(s,b,\ell))|s] + m^R(x,c,\ell) \right\}$$
$$\text{s.t.} \quad c + g(s,x) + b \leq y(s) + \Delta(s,b,\ell)$$

where $\Delta(s,b,\ell)$ represents the revenue from issuing ℓ units of debt in state s with beginning of period debt b, $\mathcal{B}(s,b,\ell)$ is next period's debt service level, conditional on repayment, and $m^R(x,c,\ell)$ is a preference shock (possibly identically 0). In the baseline setting, we let ℓ be the number of units of debt to be issued. In that case,

$$\mathcal{B}_{EG}(s,b,\ell) = (1-\lambda)b + \ell$$

and revenue is

$$\Delta^{UP}_{EG}(s,b,\ell) = p^{UP}(s,b,\ell)\ell$$

in the case of a UP or

$$\Delta^{DP}_{EG}(s,b,\ell) = \int_0^\ell p^{DP}(s,b,i)di$$

in the case of a DP, *pay-as-bid*, auction. Where $p^j(\cdot)$, $j = \{UP, DP\}$ denotes the aggregate bid function submitted by investors.

Optimal Bidding. In this environment, the private exogenous state, x, induces uncertainty on the amount of new debt the government will issue. As a result, investors sub-

[3]Effectively, V is the maximum between the value under repayment $V^R(\cdot)$ and some value under default $V^D(\cdot)$. In what follows, we will focus on the value under repayment as it is the one affected by the timing of moves within the auction.

mit multiple bids – a downward sloping individual demand. For each realization of x, let $\ell(s, b, x; p)$ denote the total quantity issued in an auction, a point in the aggregate demand curve, evaluated at the marginal price $P_c(s, b, x; p)$.

As derived in Alves Monteiro and Fourakis (2023b), under a UP, optimality requires that, for every choice of ℓ:[4]

$$p^{UP}(s, b, \ell) = \delta \mathbb{E}[Q(s', \mathcal{B}_{EG}(s, b, \ell))|s]$$

for the UP and

$$p^{DP}(s, b, \ell) = \delta \mathbb{E}[Q(s', \mathcal{B}_{EG}(s, b, \ell^\star(s, x, b)))|s, \ell^\star(s, x, b) \geq \ell]$$

for the DP.

In a DP, investors commit to pay $p(\cdot)$ regardless of how much the government borrows within the auction. In order to break even in expectation, the bid price must be equal to the expected value of debt conditional on the bid being accepted. That is, investors bid according to their expectation of how much the government will borrow – prices depend on investors' beliefs about the government's borrowing distribution. In a UP, however, investors bid the unconditional expectation of the value of the asset.

Definition 1 (Conditional EG Equilibrium)**.** *Given the auction protocol and continuation values for the government and for the lender, V and Q, a Conditional EG Equilibrium consists of bid functions, p and policy rules $\{b', \ell, P_c\}$, that satisfy the following conditions:*

1. *The bid function satisfies ex-ante zero profits for investors, given policy rules;*
2. *The policy rules solve the government's problem, given the bid function;*
3. *The auction clears, given the bid function and policy rules.*

[4]Note that different values for $\ell = \ell(s, b, x)$ emerge for different realizations of x in $\mathcal{X}$.

4 Curvature and Multiplicity

As stated in the previous section, equilibrium bids in a DP solve:

$$p(s,b,\ell) = \delta\mathbb{E}[Q(s',\mathcal{B}_{EG}(s,b,\ell^\star(s,x,b)))|s,\ell^\star(s,x,b) \geq \ell]$$

That is, investors bid according to their expectation of how much the government will borrow – prices depend on investors' beliefs about the government's borrowing distribution. It follows that a new type of static multiplicity can emerge. In particular one can think of equilibria where:

1. Lenders expect high debt issuance and therefore bid relatively low at the first few increments. Because of this, consumption is relatively low at medium levels of debt, and the marginal benefit of issuing additional units of debt is relatively high. As a result, the government issues significantly more debt (beyond medium levels), justifying lenders' beliefs that the government would issue a lot of debt.

2. Lenders expect low debt issuance and therefore bid relatively high at the first few increments. Because of this, consumption is relatively high at medium levels of debt, and the marginal benefit of issuing additional units of debt is relatively low. As a result, the government rarely issues much more debt (beyond medium levels), justifying lenders' beliefs that the government would issue not so much debt.

This multiplicity contrasts with the unique equilibrium under a UP, where equilibrium bids solve:

$$p(s,b,\ell) = \delta\mathbb{E}[Q(s',\mathcal{B}_{EG}(s,b,\ell))|s]$$

In this setting, bids that are accepted with a strictly positive probability are uniquely pinned down, regardless of investors expectations. It is simple to observe that even this weaker pointwise conditional uniqueness does not hold for the DP, since in that case bids at ℓ depend on the entire distribution of borrowing choices at least as high as ℓ.

4.1 Multiplicity: An Example

For simplicity, we use an extreme example in a simple setting. Consider a model where the government only issues debt, with one period maturity, in the first period, and from then on is in autarky. As such, it follows that $b' = \ell(s, x, b)$. Further, let the endowment be deterministic and equal to y and assume all preference shocks are identically 0 in every period. Finally, if the government defaults then it receives a random outside option v^d.

Let us consider the following parameterization: $u(x) = \log(x)$, $y = 1.1$, $b_0 = 0$, $\beta = 0.9$, $\delta = 0.99$, $v^d \sim U\big(\log(0.01), \log(0.7)\big)$ and $g(s, x) = g(x) \sim U(0, 1)$.

We follow a strategy similar to that in Aguiar and Amador (2020) and posit that under a DP there are at least two types of equilibria: one where the government borrows small amounts and one where it borrows large amounts.

Low borrowing equilibrium. We start by imposing the risk-free price $p(\ell) = Q(\ell) = \delta$ for $b' \leq \underline{b}$, where $\underline{b} : u(y - \underline{b}) = \overline{v}$. Note that the condition that characterizes $\underline{b}$ is such that the government is indifferent between repayment and defaulting if the realized outside option is the highest possible one. As such, $\underline{b}$ is the upper bound of the safe zone, where default never occurs. Given this aggregate bid function we solve the government's problem imposing $b' \leq \underline{b}$. This yields the same borrowing policy function as in the UP for $x \leq x_S$ and $b' = \underline{b}$ for $x > x_S$ where $x_S = min_x\{x : \ell(x) = \underline{b}\}$.

We then solve for the equilibrium prices and borrowing decisions for $x > x_S$, without restricting the borrowing decision to the safe zone, taking into account the prices and borrowing decisions achieved for $x \leq x_S$. Lastly we check if indeed the government prefers to stay in the safe zone.

In order to have investors bidding the risk free price it must be that borrowing is limited to the safe zone: $F(\cdot)$, the probability of repayment, equals 1 for all values of $\ell(x)$. Recall the bid function:

$$p(\ell) = \delta \mathbb{E}_x \big[F\big(u(y - b(x))\big) | \ell(x) > \ell\big] = \delta$$

The government's optimality condition simplifies to:

$$\frac{\mathbb{E}[Q(\ell)|\ell(x) > \ell]}{y + \Delta(b') - b_0 - g} = \beta \, \frac{F(log(y - b'))}{y - b'} \iff y - b' = \frac{\beta}{\delta}(y + \Delta(b') - b_0 - g)$$

$$\iff y - b' = \frac{\beta}{\delta}(y + \delta b' - b_0 - g)$$

$$\iff b' = \frac{y - \frac{\beta}{\delta}(y - b_0 - g)}{1 + \beta}$$

Which is valid as long as $b' < \underline{b}$ as otherwise the price would not be δ. As such, we have:

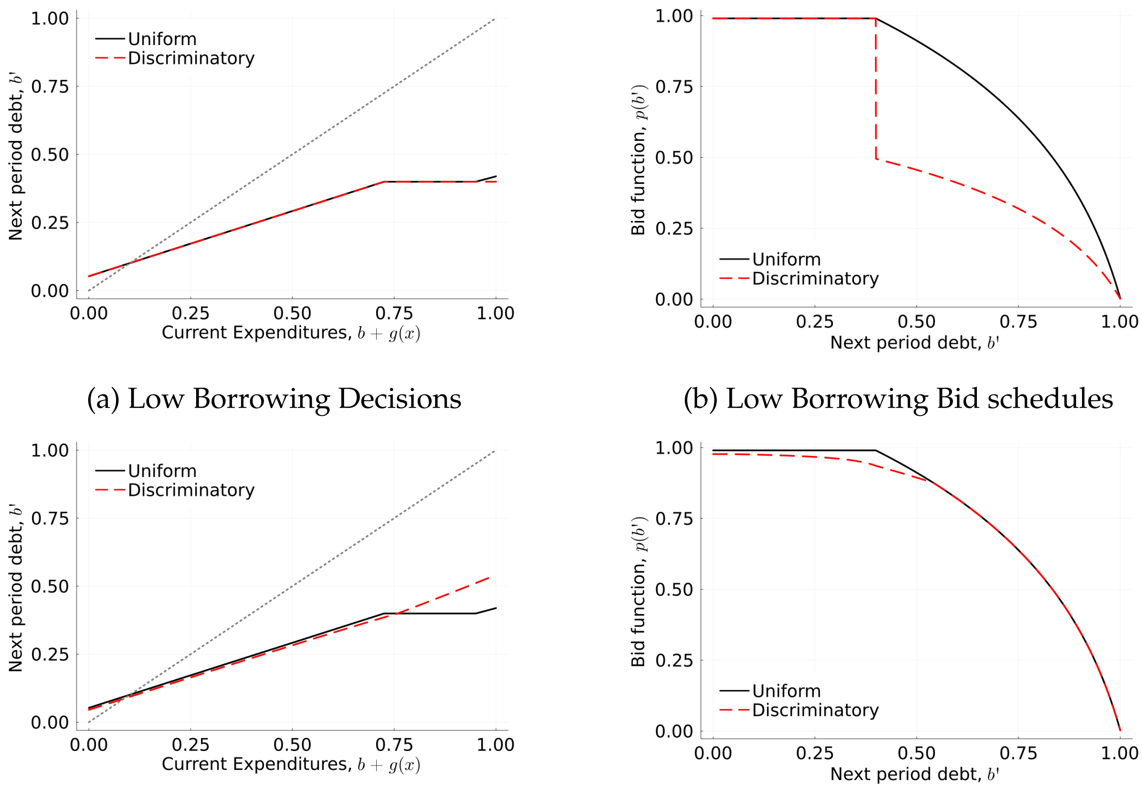
$$b' = \min\left\{\frac{y - \frac{\beta}{\delta}(y - b_0 - \theta)}{1 + \beta}, \underline{b}\right\}$$

This is an equilibrium if the price is such that the government does not borrow past $\underline{b}$. One possible schedule is $p(b') = 0$, for $b' \geq \bar{b}$. With this schedule, revenue is flat after $\underline{b}$ and the government does not borrow out of the safe zone. Note that, given the borrowing decisions, the government does not default as it never leaves the safe zone. This in turn is consistent with the price schedule.

This equilibrium hinges on the off-equilibrium price schedule: prices are such that the government does not borrow past the safe zone, and not necessarily equal to zero, as shown in Figure 1.

High borrowing equilibrium. We solve for an equilibrium numerically using discretized grids for g and b.

In this equilibrium, investors anticipate that the government will borrow past the safe zone and bid below the risk free price. Given the prices, the government borrows past the safe zone. Therefore we have constructed both a low borrowing equilibrium and a high borrowing equilibrium, as we conjectured might be possible, showcasing the potential for multiplicity.



(a) Low Borrowing Decisions

(b) Low Borrowing Bid schedules

(c) High Borrowing Decisions

(d) High Borrowing Bid schedules

Figure 1: High and Low Borrowing Equilibria, comparison of UP and DP

4.2 Uniqueness

Two features of the above environment were critical for generating the static multiplicity. The first was the the absence of any noise at all in the government's decision problem (all the preference shocks were identically 0). Because of this, we could simply choose values for the bid function at high debt levels such that those debt levels were never chosen. The second was curvature in the flow utility function. We conjecture that when the government is not too risk averse and there is nonzero noise in the government's decision problem, the Conditional EG Equilibrium is unique for the DP.

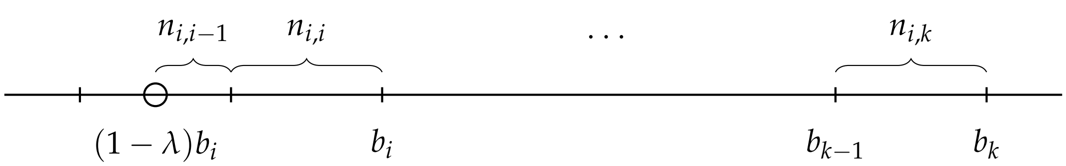
For the case of risk neutrality, we explicitly prove this under the general environment described in section 3. For the purposes of this proof, we additionally assume that there are N possible values of b. Furthermore, we suppose that the preference shocks are such that if V_i and V_j are the fundamental values (net of current preference shocks) associated with two feasible choices i and j, then the likelihood ratio of those choices is a strictly monotone increasing function of $V_i - V_j$:

$$\frac{\pi_i}{\pi_j} = f(V_i - V_j) \qquad f'(x) > 0 \qquad \lim_{x \downarrow -\infty} f(x) = 0 \qquad \lim_{x \uparrow +\infty} f(x) = \infty$$

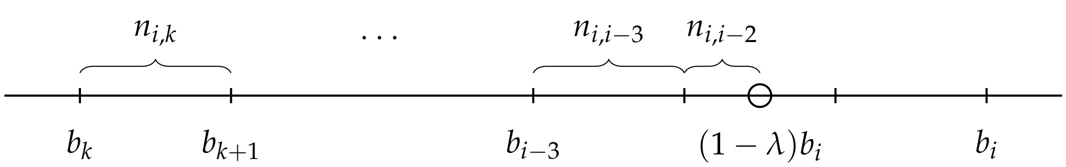
Let us first define notation for what follows. Define $n_{i,k}$ as the debt increment from b_{k-1} to b_k when debt service is moving from b_i to b_k:

$$n_{i,k} = \begin{cases} b_k - b_{k-1} & b_{k-1} > (1-\lambda)b_i \vee b_{k+1} < (1-\lambda)b_i \\ b_k - (1-\lambda)b_i & b_{k-1} \leq (1-\lambda)b_i \vee b_{k+1} \geq (1-\lambda)b_i \end{cases}$$

The first case gives us the standard increment when issuing or buying back debt. The second case deals with the first variable increment from $(1-\lambda)b_i$. Below, Figure 2 shows a graphical representation of the detailed increments.



(a) Issuance of debt $\ell = b_k - (1-\lambda)b_i$



(b) Buyback of debt $\ell = b_k - (1-\lambda)b_i$

Figure 2: Debt increments

Next we define the first and last increments for which the government is collecting revenue from an auction, respectively, $j_{start}(i,k)$ and $j_{end}(i,k)$. Set $j_{start}(i,k)$ by:

$$j_{start}(i,k) = \begin{cases} \max\{j \in \{1,...,N | b_{j-1} \leq (1-\lambda)b_i\}\} & b_k \geq (1-\lambda)b_i \\ k & b_k < (1-\lambda)b_i \end{cases}$$

and set $j_{end}(i,k)$ by:

$$j_{end}(i,k) = \begin{cases} k & b_k \geq (1-\lambda)b_i \\ \min\{j \in \{1,...,N | b_{j+1} \geq (1-\lambda)b_i\} & b_k < (1-\lambda)b_i \end{cases}$$

Consumption when choosing $\ell = b_k - (1-\lambda)b_i$ is then:

$$c_{i,k}(s,x) = y(s) - g(s,x) - b_i + \sum_{j=j_{start}(i,k)}^{j_{end}(i,k)} p_{i,j}(s) n_{i,j}$$

From here on, we suppress the dependence of everything on s and x.

Theorem 1 (Uniqueness)**.** *When the utility function is linear, the Conditional EG Equilibrium for the DP is unique.*

Proof: We first prove that the bids to buy debt (and the bids to sell it back) are unique. First, note that the bid to buy the last increment of debt, i.e. the increment yielding $b' = b_N$ must have $p_{i,N} = \delta Q_N$. If this bid is accepted by the government, then total debt must be b_N, and therefore the continuation value of lenders must be Q_N. When issuing debt, the difference in value between choice $k < N$ and choice $k+1$, $V_{i,k} - V_{i,k+1}$, is

$$\Big(y - g - b_i + \sum_{j=j_{start}(i,k)}^{k} p_{i,j} n_{i,j} + \beta V_k\Big) - \Big(y - g - b_i + \sum_{j=j_{start}(i,k+1)}^{k+1} p_{i,j} n_{i,j} + \beta V_{k+1}\Big).$$

Since $j_{start}(i,k) = j_{start}(i,k+1)$, this immediately simplifies to

$$V_{i,k} - V_{i,k+1} = \beta(V_k - V_{k+1}) - p_{i,k+1} n_{i,k+1}. \tag{1}$$

For $k = N-1$, this yields

$$V_{i,N-1} - V_{i,N} = \beta(V_{N-1} - V_N) - \delta Q_N n_{i,N}.$$

Then by assumption, this yields the ratio $\frac{\pi_{i,N-1}}{\pi_{i,N}}$ in terms of primitives.

Now note that bids $p_{i,k}$ must satisfy

$$p_{i,k} = \delta \frac{\sum_{j=k}^{N} \pi_{i,j} Q_j}{\sum_{j=k}^{N} \pi_{i,j}} = \frac{\sum_{j=k}^{N} \frac{\pi_{i,j}}{\pi_{i,k}} Q_j}{\sum_{j=k}^{N} \frac{\pi_{i,j}}{\pi_{i,k}}} = \frac{Q_k + \sum_{j=k+1}^{N} \frac{\pi_{i,j}}{\pi_{i,k}} Q_j}{1 + \sum_{j=k+1}^{N} \frac{\pi_{i,j}}{\pi_{i,k}}}. \tag{2}$$

Then $p_{i,k}$, the bid for the increment leading to b_k is known whenever $\left\{ \frac{\pi_{i,j-1}}{\pi_{i,j}} \right\}_{j \in \{k+1,\ldots,N\}}$ are known, since the numbers $\left\{ \frac{\pi_{i,j}}{\pi_{i,k}} \right\}_{j \in \{k+1,\ldots,N\}}$ can be calculated simply by taking successive products of the first sequence of likelihood ratios. Since $\frac{\pi_{i,N-1}}{\pi_{i,N}}$ is known, equation (2) pins down $p_{i,N-1}$.

Now, for $k < N - 1$, suppose that $\left\{ \frac{\pi_{i,j}}{\pi_{i,j+1}} \right\}_{j \in \{k+1,\ldots,N-1\}}$ and $p_{i,k+1}$ are known in terms of primitives. Then, we may use equation (1) to calculate $V_{i,k} - V_{i,k+1}$, which uniquely determines the ratio $\frac{\pi_{i,k}}{\pi_{i,k+1}}$ by our assumptions on f. Then we know the list $\left\{ \frac{\pi_{i,j-1}}{\pi_{i,j}} \right\}_{j \in \{k+1,\ldots,N\}}$. With this sequence of likelihood ratios, equation (2) pins down the bid $p_{i,k}$ in terms of primitives. Then, by induction, the list $\left\{ \frac{\pi_{i,j}}{\pi_{i,j-1}} \right\}_{j \in \{k+1,\ldots,N\}}$ and the bids $p_{i,k}$ with $b_k \geq (1 - \lambda) b_i$ are uniquely determined by fundamentals.

A similar construction (starting at b_1 and working up to the last b_k such that $b_k \leq (1 - \lambda) b_i$) allows us to show that the bids for buybacks are also uniquely determined by fundamentals. Since ex-ante values and choice probabilities are only functions of the continuation values, the current states, and the value of auction revenue at each possible choice, this immediately implies that the Conditional EG Equilibrium is unique. ■

In the above proof, a key property was that the marginal benefit of issuing an additional increment of debt was independent of the amount of auction revenue collected so far. Or that the marginal cost of buying back an additional increment of debt was independent of the cost incurred so far. We conjecture that it takes a substantial amount of risk aversion to generate this multiplicity, and for low risk aversion, we should still expect uniqueness. An equally important property involved the existence and nature of the noise in the government's decision problem. These allowed us to pin down likelihood ratios of choosing one level of debt relative to another, which effectively pinned down the unique price for each possible level of debt.

5 Calvo Multiplicity

Now, we consider reframing the government's decision problem following Calvo (1988). Specifically, we assume that the government's choice variable is the amount of revenue to be collected at the auction, rather than the amount of debt to be issued. The environment presented next is close to the one in Ayres et al. (2023).

Government. As before, let there be some continuation value for the government $V(s', b')$ and some continuation value for the lender $Q(s', b')$.

Under repayment, the government solves

$$V^R(s, x, b) = \max_{c,\ell} \left\{ u(c) + \beta \mathbb{E}[V(s', \mathcal{B}(s, b, \ell))|s] + m^R(x, c, \ell) \right\}$$
$$\text{s.t.} \quad c + g(s, x) + b \leq y(s) + \Delta(s, b, \ell)$$

In this case, auction revenue for both protocols is simply ℓ:

$$\Delta_{CA}(s, b, \ell) = \ell$$

This is because all units of debt in this case are issued at primary market price 1. This is engineered by adjusting the coupon rate on newly issued units of debt. Issuing a unit of debt with coupon κ adds κ units of debt service. The next period debt service level becomes

$$\mathcal{B}_{CA}^{UP}(s, b, \ell) = (1 - \lambda)b + \kappa^{UP}(s, b, \ell)\ell$$

for the UP, or

$$\mathcal{B}_{CA}^{DP}(s, b, \ell) = (1 - \lambda)b + \int_0^\ell \kappa^{DP}(s, b, i)di$$

for the DP.

Optimal Bidding. Bidding is effectively on coupon rates $\kappa^j(s, b, \ell)$, $j = \{\text{UP}, \text{DP}\}$.

First notice that we may write the functional equation for the value of debt for a bond with maturity λ and arbitrary coupon κ

$$Q(s,b|\kappa) = \mathbb{E}[(1-d')(\kappa + \delta(1-\lambda)Q(s',b'|\kappa)) + d'Q^D(s',b|\kappa)|s]$$

It is almost immediate from this definition that $Q(s,b|\kappa)$ is homogeneous of degree 1 in κ whenever $Q^D(s,b|\kappa)$ is too. We assume that $Q^D(s,b|\kappa)$ is indeed homogeneous of degree 1 in κ.[5] Then:

$$Q(s,b|\kappa) = \kappa Q(s,b|1)$$

For the UP, equilibrium bids solve

$$1 = \delta\kappa^{UP}(s,b,\ell)\mathbb{E}[Q(s',\mathcal{B}^{UP}_{CA}(s,b,\ell))|s] \tag{3}$$

whether for the DP, equilibrium bids solve

$$1 = \delta\kappa^{DP}(s,b,\ell)\mathbb{E}[Q(s',\mathcal{B}^{DP}_{CA}(s,b,\ell^\star(s,x,b)))|s,\ell^\star(s,x,b) \geq \ell] \tag{4}$$

The bond price on issuance – the left-hand-side of equations (3) and (4) – is normalized to one. Then, for each issuance ℓ, bids, $\kappa^j(\cdot)$, $j = \{UP, DP\}$, are such that the expected return to investors equals one.

Definition 2 (Conditional Calvo Equilibrium)**.** *Given the auction protocol and continuation values for the government and for the lender, V and Q, a Conditional Calvo Equilibrium consists of bid functions, κ and policy rules $\{b', \ell, P_c\}$, that satisfy the following conditions:*

1. *The bid function satisfies ex-ante zero profits for investors, given policy rules;*
2. *The policy rules solve the government's problem, given the bid function;*
3. *The auction clears, given the bid function and policy rules.*

[5] Refer to Ayres et al. (2023) for a proof of the homogeneity properties of the price function.

5.1 Discriminatory Auctions Eliminate Calvo Multiplicity

It is worth noting that different auction protocols generate different revenue curves. The UP generates a familiar looking Laffer curve for bond issuances: at first bond revenue increases as the government issues more debt, but then revenue decreases as decreases in price more than offset the increases in the amount of debt issued. Under a DP, however, revenue is always non-decreasing. Even the marginal price is indeed decreasing in the quantity issued, accepting additional, lower bids always increases total proceeds. Figure 3 illustrates revenue from a bond issuance under the UP and the DP.

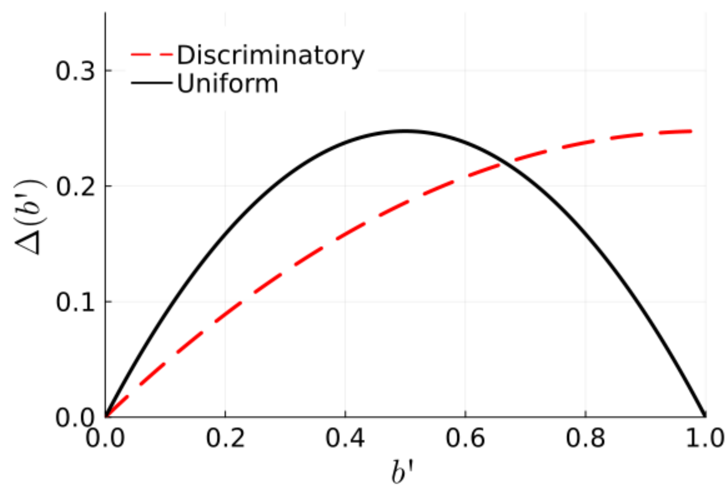


Figure 3: Revenue under discriminatory and uniform auctions

The fact that, under a DP, revenue is monotone non-decreasing indicates that, even if the government is choosing revenue instead of debt at maturity, given a revenue curve, the choice is unique (except possibly once the bid function reaches its lower bound of 0). This contrasts with the Laffer curve in the UP, where each revenue can be attained with multiple levels of debt.

For what follows, with some abuse of the term, we call the pair $(c(s,x,b), \ell(s,x,b))$ an "equilibrium allocation." We use this language because, given (s,x,b), (c,ℓ) fully characterize the lifetime value for the borrowing country and the lender. Furthermore, we assume that $Q(s',b') > 0$, in order to ensure that the Laffer Curve under the DP is strictly monotone.[6] The gist of the next theorem is that it exploits this strict monotonicity (and the

[6]In practice, this is a relatively weak assumption that is satisfied whenever there is just $\epsilon > 0$ recovery

fact that it guarantees a well defined inverse) in order to show an equivalence between a problem in which debt is the choice variable and one in which revenue is the choice variable. That inversion is used to directly construct the elements of one kind of equilibrium that are absent from the other.

Theorem 2 (Discriminatory auctions eliminate Calvo multiplicity)**.** *For the DP, the set of Calvo Equilibrium Allocations and the set of Eaton-Gersovitz Equilibrium Allocations are identical.*

Proof: Let $V(s', b')$ and $Q(s', b')$ be given. Suppose that the allocation $c^\star(s, x, b), \ell^\star(s, x, b)$ and bids $\kappa^{DP}(s, b, \ell)$ form a Calvo equilibrium.

For any ℓ_{CA}, set $\hat{\ell}_{EG}(\ell_{CA})$ by:

$$\hat{\ell}_{EG}(\ell_{CA}) = \int_0^{\ell_{CA}} \kappa^{DP}(s, b, i) di$$

Then, by definition, we will have

$$\mathcal{B}^{DP}_{EG}(s, b, \hat{\ell}_{EG}(\ell_{CA})) = \mathcal{B}^{DP}_{CA}(s, b, \ell_{CA})$$

for any ℓ_{CA}. Since $\kappa > 0$, $\hat{\ell}_{EG}(\ell_{CA})$ has a well defined inverse. Then set $p^{DP}(s, b, \ell_{EG})$ by:

$$p^{DP}(s, b, \ell_{EG}) = \frac{1}{\kappa^{DP}(s, b, \hat{\ell}^{-1}_{EG}(\ell_{EG}))}$$

Now note that revenue for the EG case satisfies:

$$\Delta_{EG}(s, b, \ell_{EG}) = \int_0^{\ell_{EG}} p^{DP}(s, b, i) di$$

Let $\ell_{EG} = \ell_{EG}(\ell_{CA})$ to obtain:

$$\Delta_{EG}(s, b, \ell_{EG}(\ell_{CA})) = \int_0^{\ell_{EG}(\ell_{CA})} p^{DP}(s, b, i) di$$

after default for arbitrarily small ϵ or if there are some states of the world s where repayment is certain for all values of b'.

Now set $j = \hat{\ell}_{EG}^{-1}(i)$ (equivalently $\hat{\ell}_{EG}(j) = i$) to obtain:

$$\Delta_{EG}(s,b,\ell_{EG}(\ell_{CA})) = \int_0^{\ell_{CA}} p^{DP}(s,b,\hat{\ell}_{EG}(j))\hat{\ell}'_{EG}(j)dj$$

Since $\hat{\ell}'_{EG}(\ell) = \kappa^{DP}(s,b,\ell)$, this becomes:

$$\Delta_{EG}(s,b,\ell_{EG}(\ell_{CA})) = \int_0^{\ell_{CA}} p^{DP}(s,b,\hat{\ell}_{EG}(j))\kappa^{DP}(s,b,j)dj$$

The integrand is now uniformly 1 by the definition of $p^{DP}(s,b,\ell_{EG})$, so this is simply:

$$\Delta_{EG}(s,b,\ell_{EG}(\ell_{CA})) = \ell_{CA}$$

Therefore, auction revenue from choosing $\ell_{EG}(\ell_{CA})$ in the Eaton-Gersovitz setting is identical to auction revenue from choosing ℓ_{CA} in the Calvo setting. It follows that the feasible set of borrowing and consumption values in the Eaton-Gersovitz setting are identical to their counterparts in the Calvo setting. Since $(c^\star(s,x,b), \ell^\star(s,x,b))$ must solve the government's maximization problem to be an equilibrium in the Calvo setting and the feasible set of consumption values and continuation values are identical in the Eaton-Gersovitz setting, $(c^\star(s,x,b), \ell_{EG}(\ell^\star(s,x,b)))$ must be the solution to the government's problem. All that remains to confirm is that $p^{DP}(s,b,\ell_{EG})$ as defined above is indeed an equilibrium bid schedule. Since $\kappa^{DP}(s,b,\ell_{CA})$ is an equilibrium bid, it must satisfy:

$$1 = \delta\kappa^{DP}(s,b,\ell)E[Q(s',\mathcal{B}_{CA}^{DP}(s,b,\ell^\star(s,x,b)))|s,\ell^\star(s,x,b) \geq \ell]$$

And therefore:

$$p^{DP}(s,b,\ell_{EG}) = \delta E[Q(s',\mathcal{B}_{CA}^{DP}(s,b,\ell^\star(s,x,b)))|s,\ell^\star(s,x,b) \geq \hat{\ell}_{EG}^{-1}(\ell_{EG})]$$

This is equivalent to:

$$p^{DP}(s,b,\ell_{EG}) = \delta E[Q(s',\mathcal{B}_{CA}^{DP}(s,b,\hat{\ell}_{EG}(\ell^\star(s,x,b))))|s,\hat{\ell}_{EG}(\ell^\star(s,x,b)) \geq \ell_{EG}]$$

Since $\ell^\star_{EG}(s,x,b) = \hat{\ell}_{EG}(\ell^\star(s,x,b))$ when the government faces these bids, it must be that:

$$p^{DP}(s,b,\ell_{EG}) = \delta E[Q(s', \mathcal{B}^{DP}_{EG}(s,b,\ell^\star_{EG}(s,x,b)))|s, \ell^\star_{EG}(s,x,b) \geq \ell_{EG}]$$

This completes the proof that every Calvo equilibrium is also an Eaton-Gersovitz equilibrium.

The proof that goes the other way is almost identical. Suppose that bids $p^{DP}(s,b,\ell)$ and the allocation $(c^\star(s,x,b), \ell^\star(s,x,b))$ form an Eaton-Gersovitz equilibrium. Now set $\hat{\ell}_{CA}(\ell_{EG})$ by:

$$\hat{\ell}_{CA}(\ell_{EG}) = \int_0^{\ell_{EG}} p^{DP}(s,b,i)di$$

Then it is immediate that auction revenue at $\ell_{CA} = \hat{\ell}_{CA}(\ell_{EG})$ is:

$$\Delta_{CA}(s,b,\hat{\ell}_{CA}(\ell_{EG})) = \int_0^{\ell_{EG}} p^{DP}(s,b,i)di$$

Therefore, choosing $\ell_{CA} = \hat{\ell}_{CA}(\ell_{EG})$ yields the same consumption value as choosing ℓ_{EG} would have for the Eaton-Gersovitz setting. Next, we need to show that the debt service values are also identical. To do this, start by defining the bids for the Calvo setting as:

$$\kappa^{DP}(s,b,\ell_{CA}) = \frac{1}{p^{DP}(s,b,\hat{\ell}^{-1}_{CA}(\ell_{CA}))}$$

evaluate $\mathcal{B}^{DP}_{CA}(s,b,\ell_{CA})$ at $\ell_{CA} = \hat{\ell}_{CA}(\ell_{EG})$:

$$\mathcal{B}^{DP}_{CA}(s,b,\hat{\ell}_{CA}(\ell_{EG})) = (1-\lambda)b + \int_0^{\hat{\ell}_{CA}(\ell_{EG})} \kappa^{DP}(s,b,i)di$$

Now set $j = \hat{\ell}^{-1}_{CA}(i)$ (equivalently $\hat{\ell}_{CA}(j) = i$) to obtain:

$$\mathcal{B}^{DP}_{CA}(s,b,\hat{\ell}_{CA}(\ell_{EG})) = (1-\lambda)b + \int_0^{\ell_{EG}} \kappa^{DP}(s,b,\hat{\ell}_{CA}(j))\hat{\ell}'_{CA}(j)dj$$

Since $\hat{\ell}'_{CA}(\ell_{EG}) = p^{DP}(s,b,\ell_{EG})$, this becomes:

$$\mathcal{B}^{DP}_{CA}(s,b,\hat{\ell}_{CA}(\ell_{EG})) = (1-\lambda)b + \int_0^{\ell_{EG}} dj$$

which immediately implies:

$$\mathcal{B}^{DP}_{CA}(s,b,\hat{\ell}_{CA}(\ell_{EG})) = (1-\lambda)b + \ell_{EG} = \mathcal{B}^{DP}_{EG}(s,b,\ell_{EG})$$

Again, the feasible set of consumption and next period debt service levels faced by the government in our constructed Calvo equilibrium are identical to those faced by the government in the assumed Eaton-Gersovitz equilibrium. Therefore, the government's solution in the Calvo setting must be $(c^\star(s,x,b), \hat{\ell}_{CA}(\ell^\star(s,x,b)))$. All that is left to do is confirm that $\kappa^{DP}(s,b,\ell_{CA})$ satisfies the functional equation for bids. Since, by assumption, $p^{DP}(s,b,\ell_{EG})$ is an equilibrium bid function, we must have:

$$p^{DP}(s,b,\ell_{EG}) = \delta E[Q(s', \mathcal{B}^{DP}_{EG}(s,b,\ell^\star(s,x,b)))|s, \ell^\star(s,x,b) \geq \ell_{EG}]$$

Evaluating this at $\ell_{EG} = \hat{\ell}^{-1}_{CA}(\ell_{CA})$ and substituting in using the definition of $\kappa^{DP}(s,b,\ell_{CA})$ then yields:

$$\frac{1}{\kappa^{DP}(s,b,\ell_{CA})} = \delta E[Q(s', \mathcal{B}^{DP}_{EG}(s,b,\ell^\star(s,x,b)))|s, \ell^\star(s,x,b) \geq \hat{\ell}^{-1}_{CA}(\ell_{CA})]$$

A little rearrangement and a slight change of variables in the condition on the right hand side yields:

$$1 = \delta\kappa^{DP}(s,b,\ell_{CA}) E[Q(s', \mathcal{B}^{DP}_{EG}(s,b,\ell^\star(s,x,b)))|s, \hat{\ell}_{CA}(\ell^\star(s,x,b)) \geq \ell_{CA}]$$

Since $\ell^\star_{CA}(s,x,b) = \hat{\ell}_{CA}(\ell^\star(s,x,b))$ and $\mathcal{B}^{DP}_{CA}(s,b,\hat{\ell}_{CA}(\ell_{EG})) = \mathcal{B}^{DP}_{EG}(s,b,\ell_{EG})$, this is equivalent to:

$$1 = \delta\kappa^{DP}(s,b,\ell_{CA}) E[Q(s', \mathcal{B}^{DP}_{CA}(s,b,\ell^\star_{CA}(s,x,b)))|s, \ell^\star_{CA}(s,x,b) \geq \ell_{CA}]$$

Therefore, the constructed $\kappa^{DP}(s,b,\ell_{CA})$ is an equilibrium bid function. It follows that any Eaton-Gersovitz Equilibrium is also a Calvo Equilibrium. This completes the proof. ■

The above proof required extremely few conditions on the underlying objects. The exercise was almost entirely just about showing that the feasible sets were identical, and therefore optimality in a choice immediately implied optimality in a transformation of that choice. Switching to the Calvo environment (choosing revenue, rather than debt) will never change the set of Conditional Equilibria (i.e. equilibria today given continuation values).

6 Discussion

Even though the DP is prone to static debt dilution (as is detailed at length in Alves Monteiro and Fourakis (2023a)), we have shown that, in the case of a risk neutral government (and with certain assumptions about the distribution of the noise), the set of Conditional Eaton-Gersovitz Equilibria contains exactly one element. We conjecture that it takes a substantial amount of risk aversion to generate this static multiplicity, and for low (but nonzero) risk aversion, we should still expect uniqueness. This conjecture is similar to the result in Stangebye (2020) that emphasizes concavity in the utility function as crucial in supporting multiplicity in an environment with long term debt. This similarity, once again, points to the parallel between an environment with one period debt and a DP and one with long term debt and a UP. In the latter there is dynamic debt dilution, over time, whereas in the first there is static debt dilution, across states. Similarly, our uniqueness result is also reminiscent of DeMarzo et al. (2023), who find a unique equilibrium with long term debt under linear preferences.

In a more general proof, that does not depend on the concavity of the utility function, nor on the properties of the continuation values V and Q, we showed that, under a DP, the Conditional Equilibrium sets under the Eaton-Gersovitz and Calvo settings are equivalent. This contrasts starkly with Conditional Equilibrium sets under a UP. As shown by Ayres et al. (2018), with the UP and a bimodal distribution for the underlying shocks that drive borrowing and the the risk of default, it is possible that switching from the Eaton-Gersovitz to the Calvo setting produces a static multiplicity (i.e. in the language of this paper, multiple Conditional Equilibria, given continuation values).

Together, these two results imply that, under a DP and a risk neutral government, the Conditional Equilibrium is unique and independent of the specification of the choice variable, be it debt quantity or the budget deficit.

7 Conclusion

In this paper we focused on how different auction protocols make the government more or less vulnerable to multiplicity driven by self-fulfilling crises. We first described how using the DP opens the door to another type of static multiplicity. As equilibrium bids depend on investors' beliefs about how much debt is going to be issued in a given auction, different beliefs may support different equilibria in the current period, taking as given a single set of future payoffs.

We then showed that curvature in the flow utility (as in Stangebye (2020)) and the absence of any noise in the government's decision problem is important to generate this multiplicity. In particular, for a risk neutral government that faces noise in its decision problem, we show that the equilibrium under a DP is unique.

Using a model of sovereign borrowing and default with repeated auctions, we then show that the DP eliminates the static multiplicity found in Calvo (1988). We do so by showing that the set of Conditional Equilibria in the Eaton-Gersovitz setting is equivalent to the set in the Calvo setting. This is in stark contrast with the UP that is prone to more equilibria in the latter.

If we assume that Calvo's characterization of the environment (fiscal policy decisions yield a deficit to be financed, and debt is issued until it is financed) is more accurate, our results provide a rationale for using the DP. Specifically, governments concerned about self-fulfilling expectations in the primary market for debt may be able to avoid the (typically very bad)[7] outcomes associated with the "bad equilibrium" that could arise if they were to use the UP.

[7] See for example Ayres et al. (2018) or Ayres et al. (2023)

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