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RESEARCH ARTICLE

On the Welfare Losses from External Sovereign Borrowing

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Abstract
This paper studies the losses to the citizenry when the private agents discount the future at different rates from their government. In the presence of such a disagreement, the private sector may prefer an environment in which the government is in financial autarky. Using a sequence of sovereign debt models, the paper quantifies the potential welfare losses that citizens suffer from the government's access to international bond markets. While the environment is not necessarily comprehensive, the analysis provides a counterweight to proposals that are designed to *ease* market access for sovereign borrowers.

JEL Classification F34 · F41

1 Introduction

The fact that Argentina is experiencing a fiscal crisis only two years after coming to terms with bondholders from the previous default is just the latest reminder that governments frequently borrow to the point of default. Reinhart and Rogoff (2004) refer to this phenomenon as "serial default." Rationalizing this pattern typically begins with modelling governments, and the politicians that run them, as impatient.[1]

[1] A well-established political economy literature has provided several models that generate impatient policy makers. See, for example, Alesina and Tabellini (1990) and Azzimonti (2011) for closed economy environments and Amador (2003), Aguiar and Amador (2011) and Cuadra and Sapriza (2008) for environments with external sovereign debt.

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In particular, the government's discount rate is assumed to be higher than that of international lenders. This is a pervasive feature of the political economy literature as well as the quantitative sovereign debt models.

While the government may be relatively impatient vis-à-vis the world interest rate, the sovereign debt literature is often silent on the discount rates of domestic private agents. We explore how sensitive is private sector's welfare to disagreement in discount rates between politicians and their constituents. We do so by asking a simple but stark question: at what level of disagreement would the private sector be better off if the government *had zero access to international financial markets.*

This zero external borrowing rule is an extreme version of the rich set of fiscal rules seen in practice.[2] In this paper, we focus on the costs of external public indebtedness and abstract from domestic debt, taxation, and redistribution considerations.[3] We propose an exact decomposition of the effects of external access on private sector's welfare that applies to the sovereign debt models recently used in quantitative work.[4] We identify three costs that arise when an impatient government can access international sovereign debt markets:

1. *Front-loading of expenditures* from the perspective of a more patient household, an impatient governments shifts too much spending towards the present.
2. *Excess variability of expenditures* by borrowing more in good endowment states than in bad, the government may introduce additional variability to the spending allocation.
3. *Default costs* By borrowing, the government exposes the country to a future default and its associated costs.

To explore and quantify these potential channels, the analysis begins with a simple deterministic benchmark. In this benchmark, the government faces a debt limit (where the debt limit is assumed to be such that the government does not default). Access to external debt markets allows the government to front-load the consumption allocation, and this is the only distortion relative to the household optimal path of spending. We show that this tilting of consumption in the absence of default has minimal welfare consequences for standard values of the inter-temporal elasticity of substitution: households strictly prefer that their government borrows (unless their discount factor is close to the interest rate). Hence, this simple model does not make a strong case for banning sovereign borrowing unless the households are sufficiently patient.

The simple model, however, ignores two important elements: uncertainty and the possibility of default in equilibrium. To assess the implications of these, we turn to quantitative sovereign debt models that incorporate both of these elements. In these models, which are based on an earlier contribution by Eaton and Gersovitz (1981), the government faces uncertainty with respect to its revenue or endowment, and borrows with an uncontingent bond. However, the government may choose to default, in

[2] See Lledó et al. (2017) for a survey of existing rules.

[3] Malaysia is an example where the fiscal rule includes explicit limits on external public debt, in addition to domestic.

[4] These models are based on the original contribution of Eaton and Gersovitz (1981).

which case it suffers a period of reduced output and no access to international financial markets. We begin with the early contributions of Aguiar and Gopinath (2006) and Arellano (2008).

The Aguiar–Gopinath (AG) model with transitory shocks hews closely to the original Eaton and Gersovitz (1981) model. The major difference is adding a proportional drop in the endowment, while the country is in default. We show that this model generates results strikingly similar to the back-of-the-envelope benchmark described above. The reason is primarily due to the infrequent default under this calibration, a feature of the model emphasized in the original AG paper.

Arellano (2008) introduces a richer notion of default costs. In particular, Arellano assumes a nonlinear decline in the endowment in default, with no losses for low output realizations and large declines for high output realizations. The fact that equilibrium default occurs in low-endowment states implies negligible deadweight losses as well as a higher frequency of default relative to AG. Despite the frequency of default, the fact that deadweight losses are minimal generates welfare losses similar in magnitude to AG and the benchmark calculation. However, compared to AG, the losses are more sensitive to the differences in discount factors. This result is explained by the excess variability of expenditures that market access generates. However, the magnitude of the welfare losses is only on the order of 0.2% of consumption.

We then turn to the richer environments explored by Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012). These papers introduced long-term bonds and flagged that the incentive to dilute existing bondholders plays an important role in debt dynamics and the frequency of default. We first show that despite the incentive to dilute and the potential beneficial role for fiscal rules (see Hatchondo et al. 2012), the government strictly prefers access to bond markets over financial autarky. The rule of zero access to sovereign markets is too costly to the government: it will never voluntarily shut itself out of sovereign debt markets.

Using the calibration of Chatterjee and Eyigungor (CE), we find large welfare losses for households at relatively small levels of discount rate disagreement. The reason for the sharp difference is that the simulated economy spends a significant fraction of time in the default state. Given the endowment costs due to default, this generates a large deadweight loss. We conclude that the presence of default costs in equilibrium significantly strengthens the case for banning international sovereign debt borrowing.

Our focus on the framework based on Eaton and Gersovitz (1981) ignores additional mechanisms that can affect our results. First, the models we explore in this paper do not include domestic capital and investment. It is indeed possible that an impatient government may borrow to invest (rather than consume) if the marginal product of capital is sufficiently high, and such borrowing may be beneficial to households, as it increases their future consumption.[5] Whether or not emerging

[5] A recent contribution here is Gordon and Guerron-Quintana (2018), which introduces investment to the Eaton–Gersovitz framework.

market governments borrow internationally to invest in productive domestic investment projects is subject to discussion.[6] Related to this, the models we explore here do not allow the private sector to borrow internationally – hence eliminating another potential benefit of accessing foreign debt markets.[7] We have also chosen to focus the analysis to just disagreement about one parameter: the discount rate of households and politicians. However, the government and households may disagree about their valuations of risk and their inter-temporal elasticity of substitution. Probably more importantly, the households and the government could also disagree with regard to the composition of expenditures.[8] Our framework also ignores the possibility of self-fulfilling sovereign debt crises (as in Cole and Kehoe 2000), which again exposes a borrowing country to additional default risk.

1.1 Related Literature

For a comprehensive review of the sovereign debt literature, we refer to Aguiar and Amador (2014) and Aguiar et al. (2016). That some countries seem to be serial defaulters was first documented in Reinhart and Rogoff (2004).[9] The quantitative sovereign debt literature is based on the model of Eaton and Gersovitz (1981), which assumes the government is a benevolent social planner that decides on the expenditures and the amount to borrow externally, but it is unable to commit not to default on its debt. With one-period bonds, there is no benefit for the government to commit to any fiscal rule that restricts its future borrowing and spending decisions (see Aguiar and Amador 2019). When the government has time-inconsistent preferences, Alfaro and Kanczuk (2017) discuss the role for fiscal rules within such an environment starting from zero debt. They also analyse the case where the government has standard exponentially discounted preferences, but its discount factor differs from the citizen's. With longer-duration bonds, there are potentially benefits of committing to fiscal rules even for a benevolent government, a point explored in detail in Hatchondo et al. (2012). That paper also discusses the case for fiscal rules when the private agents have a different discount factor from the government.[10] Our contribution, in relation to the last two papers, is to provide a simple benchmark exercise to quantify the losses from market access, as well as to investigate the magnitudes and the degree of disagreement across several calibrations and environments. In addition, rather than exploring different fiscal rules, we focus on the simpler question of

[6] See, for example, Gourinchas and Jeanne (2013) and Aguiar and Amador (2011).

[7] The planning representation used in Eaton and Gersovitz (1981) and subsequent literature is usually interpreted as stating that the government has sufficient instruments to control the private sector's decisions. See Jeske (2006) for a different approach.

[8] For example, the government may decide to spend resources on goods that are not as valued by the households (or just channel the external funds to the private accounts of politically connected entities).

[9] See also Amador and Phelan (2018) for a model as well as other references.

[10] See also the work of Azzimonti et al. (2016) for a closed economy counterpart of the benefits of balanced budget rules starting from any particular level of debt.

whether the citizens would prefer that their government had no access to external debt markets.[11]

2 A Simple Model of Inter-Temporal Disagreement

In this section, we set the stage for our quantitative analysis by considering a simple model that admits closed-form solutions. In addition to providing a useful reference, it allows a back-of-the-envelope calculation that demonstrates that inter-temporal disagreement in a default-free environment has limited welfare consequences quantitatively.

Consider a simple deterministic consumption-saving problem in which a potentially impatient government decides how much to consume and borrow from international financial markets. Time is infinite and continuous. There is a small open economy (SOE) which is endowed with a constant endowment flow, y. At every instant, the government decides the consumption of the representative consumer of the SOE, c_t, and finances it with the endowment plus issuing bonds with face value b_t to international financial markets. International financial markets are willing to lend to the government at a constant interest rate r up to a borrowing limit, $\overline{b}$.

The budget constraint for the SOE is:

$$\dot{b}_t = c_t + rb_t - y, \tag{1}$$

which states that the change in debt equals consumption plus interest rate payments on the debt minus the endowment.

2.1 The Government's Value

The government's preferences over consumption sequences are given by:

$$U = \int_0^\infty e^{-\rho_G t} u(c_t)\mathrm{d}t.$$

We impose that $\rho_G > r$, so that the government is more impatient than the market interest rate, and, as a result, ends up borrowing to the maximum $\overline{b}$.

A useful case, which we pursue throughout, is that of power utility: $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$ for $\sigma > 0$ and $\sigma \neq 1$ and $u(c) = \log(c)$ for $\sigma = 1$. The parameter $\sigma = \frac{u'(c)}{-cu''(c)}$ is the inter-temporal elasticity of substitution (IES).

The solution to the government's problem is straightforward. The fact that $\rho_G > r$ implies that the government borrows up to the limit. The speed at which it does this is governed by the degree of impatience and the inter-temporal elasticity of substitution (IES). In particular, while $b_t < \overline{b}$, consumption obeys the Euler equation:

[11] For other models where fiscal rules are useful, see Dovis and Kirpalani (2018) and the work of Halac and Yared (2018).

$$\frac{\dot{c}}{c} = \frac{u'(c)}{-cu''(c)}\left(\rho_{\mathrm{G}} - r\right).$$

once $b_t = \overline{b}$, then $c = y - r\overline{b}$.

With constant IES, the solution has a closed form. Specifically, starting from $b_0 = 0$, let T denote the first time $b_t = \overline{b}$. We have

$$c_t = \begin{cases} e^{\frac{\rho_{\mathrm{G}}-r}{\sigma}(T-t)}(y - r\overline{b}); & t \in [0, T] \\ y - r\overline{b}; & t > T, \end{cases} \tag{2}$$

where T is such that

$$1 - r\frac{\overline{b}}{y} = \frac{e^{rT}}{1 - r\frac{\sigma}{\rho_{\mathrm{G}}-r(1-\sigma)}\left(1 - e^{\frac{T(\rho_{\mathrm{G}}-r(1-\sigma))}{\sigma}}\right)}.$$

Let V_0 denote the associated value of the government, where the subscript zero reminds us that the initial debt is zero. For convenience, let us define the value of autarky for the government:

$$V^A = \frac{u(y)}{\rho_{\mathrm{G}}},$$

which represents the value to the government if it could not borrow nor save internationally (in which case it is constrained to equalize its expenditures to the endowment).

Given that the government can always choose not to borrow, it follows that $V_0 \geq V^A$. It is also straightforward to argue that if $\rho_{\mathrm{G}} > r$ and $\overline{b} > 0$, $V_0 > V^A$. Hence, not surprisingly, *the government finds it beneficial to have access to international markets*. As we will see below, this result survives the presence of uncertainty, potential default, and, more importantly, the introduction of debt dilution into the model.

The question of interest relates to the benefits of access to international financial markets from the perspective of the households of the SOE, to which we now turn.

2.2 The Households' Value

We assume that, in addition to the government, the SOE also contains a representative household. This household cannot borrow or save internationally, and values the expenditure flows generated by the government using the same utility flow function as the government, u. Our main assumption is that the household uses a different inter-temporal discount factor, ρ_{H}. Clearly, when $\rho_{\mathrm{H}} = \rho_{\mathrm{G}}$, the government is benevolent and agrees with the household ranking of consumption paths. However, when $\rho_{\mathrm{H}} < \rho_{\mathrm{G}}$, then we say the *government is impatient* with respect to the household. In this case, the household ranking of consumption paths disagrees from the government's.

Let W_0 denote the welfare of the household under the consumption plan chosen by the government in (2) and constant IES preferences:

$$W_0 = \begin{cases} e^{-T\rho_H}\left(\frac{1}{\rho_H} + \frac{e^{T\left(\frac{\rho_G - r}{\sigma}(1-\sigma)+\rho_H\right)}-1}{\frac{\rho_G - r}{\sigma}(1-\sigma)+\rho_H}\right)\frac{(y-r\bar{b})^{1-\sigma}}{1-\sigma}; & \text{if } \sigma \neq 1 \text{ and } \sigma > 0 \\ \frac{\rho_G - r}{\rho_H}\left(\frac{e^{-T\rho_H}-1}{\rho_H} + T\log(y - r\bar{b})\right); & \text{if } \sigma = 1. \end{cases}$$

The representative household's autarkic value is:

$$W^A = \frac{u(y)}{\rho_H}$$

Note that if $\rho_H = r$, then autarky is the optimal consumption plan from the perspective of the household. That is, if $\rho_H = r$, then $W_0 < W^A$ for any $\rho_G > r$ and $\bar{b} > 0$. In this simple environment, if the household's discount rate equals the foreign interest rates, then giving the government access to international financial markets unambiguously reduces household's welfare. The intuition is straightforward: with no access to international financial markets, consumption equals the constant endowment, which corresponds to the best allocation from the household perspective. Allowing the government the ability to distort consumption inter-temporally away from this benchmark induces a reduction in household welfare.

2.3 The Welfare Costs of Financial Market Access: A Simple Calculation

To assess the welfare costs of market access when the household and the government disagree on inter-temporal trade-offs, define $\hat{\lambda}$ as the percentage increase in the autarky consumption path that would make a household indifferent between this allocation and the allocation where the government follows its optimal borrowing plan. That is:

$$\hat{\lambda} \equiv \begin{cases} \left(\frac{W_0}{W^A}\right)^{\frac{1}{1-\sigma}} - 1; & \text{if } \sigma \neq 1 \\ e^{\rho_H(W_0 - W^A)} - 1; & \text{if } \sigma = 1, \end{cases}$$

where $\hat{\lambda}$ captures the *welfare gains from international financial market access*.

Using the previous calculations, we obtain the following result:

Lemma 1 *For $\bar{b} > 0$ and $\rho_G > r$, the welfare gains from international financial market access are strictly increasing in ρ_H, and given by*

$$\hat{\lambda} = \begin{cases} \frac{e^{rT}(\rho_G - r(1-\sigma))}{\rho_G - r\left(1 - \sigma e^{T\frac{\rho_G - r(1-\sigma)}{\sigma}}\right)}\left(\frac{\rho_H \sigma e^{T(1-\sigma)\frac{\rho_G - r}{\sigma}} + e^{-T\rho_H}(\rho_G - r)(1-\sigma)}{(\rho_G - r)(1-\sigma) + \rho_H\sigma}\right)^{\frac{1}{1-\sigma}} - 1; & \text{if } \sigma \neq 1 \text{ and } \sigma > 0 \\ \frac{\rho_G}{\rho_G - r(1 - re^{\rho_G T})} e^{(\rho_G - r)\frac{e^{-T\rho_H}-1}{\rho_H} + \rho_G T}; & \text{if } \sigma = 1. \end{cases} \tag{3}$$

Proof In the Appendices. □

As stated in the lemma, the welfare gains are strictly increasing in ρ_{H}: a more *impatient* household values more the allocation where its government borrows internationally,

At the other end, when the households are infinitely patient, that is, when $\rho_{\mathrm{H}} \to 0$, $\hat{\lambda}$ converges to:

$$\lim_{\rho_{\mathrm{H}} \to 0} \hat{\lambda} = -r\frac{\overline{b}}{y}.$$

This result is quite intuitive: in the limit, consumption converges to $y - r\overline{b}$, and hence, an infinitely patient households needs to be positively compensated with respect to its endowment path by exactly $r\overline{b}/y$. It follows as well that this value is the maximum possible loss, as $\hat{\lambda}$ is strictly increasing in ρ_{H}.

The question that concerns us is quantitative: How large are the potential losses from financial market access? And how are they related to the difference in discount factors between the households and the government?

Towards this goal, let us make the following parametric assumptions, which lie within the ballpark of the assumptions made in the quantitative sovereign debt literature. We set $\sigma = 2$, and let $r = 0.04$, representing a 4% real annual rate of return on a safe external asset. We let $\overline{b}/y = 0.25$, representing a 25% of external debt over the annual GDP. It is clear from the above expression that the limiting welfare loss is linear in the debt-to-GDP level chosen; hence, it is straightforward to compute alternative losses for different calibrations.[12]

In our first exercise, we vary ρ_{G} and calculate the value of ρ_{H} that makes the household indifferent between market access or autarky, that is, the value of ρ_{H} such that $\hat{\lambda} = 0$. The results of this exercise are summarized in Fig. 1. In this figure, the solid thick line represents combinations of the government and the household discount factors such that the household is indifferent between market access and autarky.

Figure 1 shows that the discount rate that makes households indifferent between autarky and financial market access is (i) very close to the world interest rate and (ii) almost insensitive to the discount rate of the government. Even when the discount rate of the government is 0.80 (which is equivalent to an annual discount factor of 0.45), *a household with a discount rate higher than 0.0524 (that is, a discount factor below 0.95) strictly prefers financial market access to autarky*.[13]

The second exercise highlights the magnitudes of the gain/losses. For this, we let $\rho_{\mathrm{G}} = 0.20$, a value commonly used in the quantitative literature (which we discuss below). We then compute $\hat{\lambda}$ using the previous parameter values, while varying the household discount rate, ρ_{H}.

[12] We derived this calculation in continuous time, but given the deterministic nature of the environment, the same quantitative results hold (approximately) in a discrete time version.

[13] To compute the implicit annual discount factor, we compound the annual discount rate for one unit of time: $\beta_{\mathrm{G}} = e^{-\rho_{\mathrm{G}}}$.

Figure 2 shows the associated values of $\hat{\lambda}$ as a function of ρ_{H}. Note that when ρ_{H} is close to zero, the welfare gains are negative (that is, the households prefer autarky) and close to $-r\overline{b}/y = -0.01$, as expected. That is, for ρ_{H} close to zero, the households would be willing to reduce their consumption by 1% in order for the government not to access international financial markets. Note also that very close to the market discount rate, the welfare gains turn positive and become large. For example, for a household discount rate of 10%, the welfare gains are above 1% of consumption.

Let us now briefly summarize the results of the simple benchmark exercise. First, the welfare costs of financial market access are bounded above by $r\overline{b}/y$. Second, for parameter values close to those typically assumed in the quantitative sovereign debt literature, the discount rate that keeps households indifferent between financial market access and autarky remains numerically close to the market risk-free interest rate. As a result, households which are only slightly more impatient than the markets strictly prefer that their governments maintain financial market access in order to front-load consumption and borrow the maximum amount. Finally, the welfare gains from having access to financial markets can be potentially large, as the household discount rate increases above the market rate.

Our simple back-of-the-envelope exercise suggests that allowing governments to borrow internationally is beneficial for their citizens, even when such governments may be extremely impatient. However, this exercise has ignored the role of shocks, the possibility of not paying the international debts, and the potential costs of default. We now show that incorporating such elements makes the case for banning international financial market access much stronger.

3 The Canonical Sovereign Debt Model

The quantitative sovereign debt literature is based on the model of Eaton and Gersovitz (1981).[14] The models in this area incorporate stochastic endowment shocks, defaultable but otherwise non-contingent bonds, the possibility of default occurring in equilibrium, the existence of default costs, and the possibility of re-entry to financial markets after a default.[15]

We now introduce the benchmark environment. Time is discrete. There is a small open economy, with a government that can access international financial markets. Let $s \in S$ denote the exogenous state and s^t represent the history of the state realizations. The state evolves according to a probability function given by $\pi(s'|s)$. In every period, the country receives an endowment, $y(s)$, in units of the single consumption good.

[14] Early contributions to the quantitative literature are Aguiar and Gopinath (2006), Arellano (2008), Hamann (2002) and Yue (2010).

[15] The model has also been extended to incorporate bargaining among creditors and the sovereign after a default, production, and risk-averse lenders.

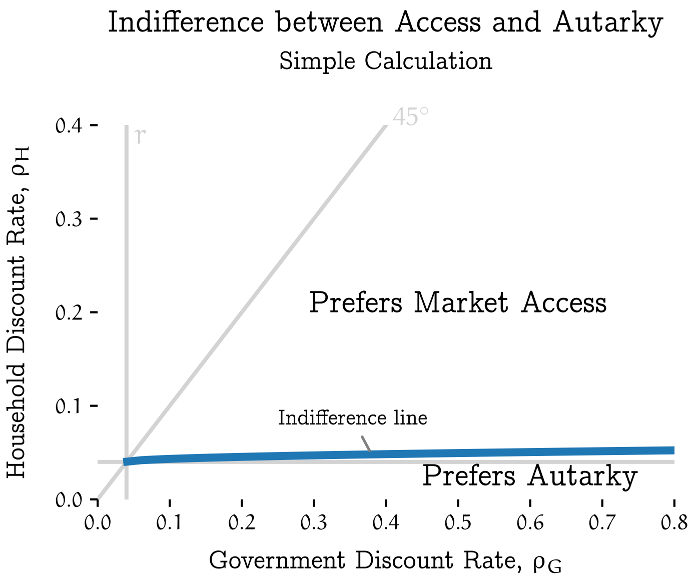

Fig. 1 Values of ρ_H above the solid thick line represent values for which the households strictly prefer access to financial markets over autarky. Values below are those where the opposite is preferred. The indifference points are captured by the solid thick line. The vertical and horizontal lines represent the market discount rate

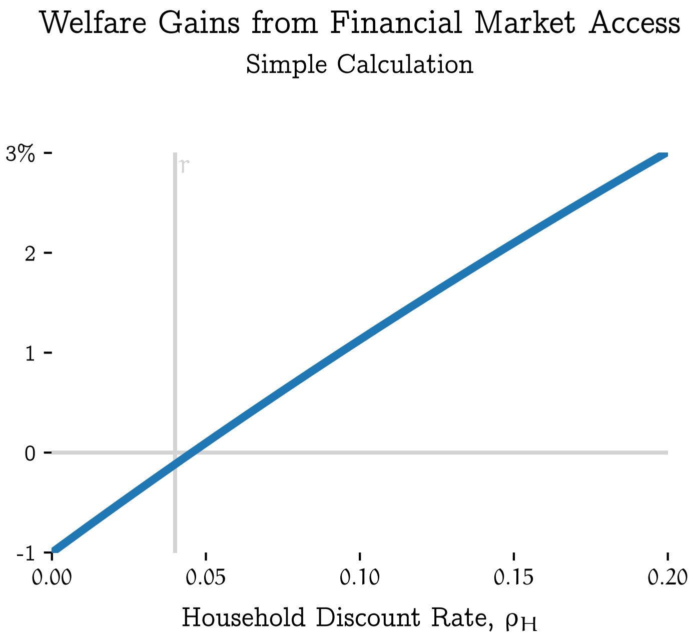

Fig. 2 The solid thick line represents the value of $\hat{\lambda}$ (y axis) as a function of the household discount rates ρ_H (x axis) assuming $\rho_G = 0.20$. The vertical line represents the market discount rate, $r = 0.04$. The horizontal solid line represents $\hat{\lambda} = 0$

As long as it has access to international credit markets, the government can issue bonds, each of which is a promise to deliver an exponentially declining coupon, δ^t, t periods from its issuance date.[16] Note that this implies, absent issuances or repurchases, that bonds at time t promise a stream of payments that are equivalent to δ times that of bonds at time $t-1$. Hence, if b_t is the face value of bonds at time t, net issuances are $b_t - \delta b_{t-1}$.

The government can, at any time, choose to default. In such case, the outstanding bonds lose all value and the government enters a (temporary) exclusion period. While it is excluded from credit markets, the government cannot issue bonds or save, and the economy's endowment is reduced to $y^D(s) \leq y(s)$. Once in financial autarky, the government may re-enter the financial markets (starting with zero debt) at which point the endowment process reverts back to $y(s)$ and the government regains its ability to trade financial instruments. This re-entry after a default occurs with a constant Poisson probability θ.

In every period, the government decides whether to default or not, how much debt to issue (if it has access to financial markets), and the level of its expenditures. We assume that the government has preferences given by

$$\mathbb{E} \sum_{t=0}^{\infty} (\beta_{\mathrm{G}})^t u(c_t),$$

where $\beta_{\mathrm{G}} \equiv e^{-\rho_{\mathrm{G}}}$ and the expectation is over the Markov process s conditional on the initial state.

As usual in the literature, we narrow attention to Markov perfect equilibria. The payoff relevant state variables are s, whether the government is in autarky or not, and its current level of outstanding debt.

For the case of a government that enters the period in good credit standing, and decides to repay its debts, its value is given by the solution to the following problem:

$$V(b,s) = \max_{b' \leq \overline{B}} \left\{ u(c) + \beta_{\mathrm{G}} \sum_{s'|s} \pi(s'|s) \max\{ V(b',s'), \underline{V}(s') \} \right\}$$

subject to:

$$c = y(s) - b + q(b',s)(b' - \delta b),$$

where b denotes the current level of outstanding debt; $q(b',s)$, the price of the bonds; and $\underline{V}(s')$, the value of default. The value of $\overline{B}$ is assumed to be sufficiently large that its only role is to rule out Ponzi schemes. Note that we have incorporated the next period default decision into the government problem. We let $\mathcal{B}(b,s)$ denote the debt policy function that solves this problem.

In case of default, the payoff to the government is:

[16] This follows Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012).

$$\underline{V}(s) = u(y^D(s)) + \beta_G \sum_{s'|s} \pi(s'|s)\big(\theta V(0, s') + (1-\theta)\underline{V}(s')\big).$$

Finally, we need to specify how the financial markets value the bonds. We assume that the financial markets are risk neutral, and discount inter-temporal flows at a rate R. The bond price is then given by the following break-even condition:

$$q(b, s) = \frac{1}{R} \sum_{s'|s} \pi(s'|s) \mathbb{1}_{\{V(b,s') \geq \underline{V}(s')\}} \big(1 + \delta q(\mathcal{B}(b, s'), s')\big),$$

where $\mathbb{1}_{\{x\}}$ is an indicator function that returns 1 if x is true and $\mathcal{B}(b, s')$ was defined above to be the government's debt issuance policy function.

Definition 1 A Markov equilibrium is then set of functions $V, \underline{V}, q, \mathcal{B}$ such that (i) V and $\underline{V}$ are fixed points of the government's Bellman equations; (ii) $\mathcal{B}$ is a solution of the government's repayment problem; and (iii) q satisfies the break-even condition for the financial markets.

The reference allocation is autarky.[17] We define the autarky welfare (starting from zero debt) for the government as:

$$V^A(s) = u(y(s)) + \beta_G \sum_{s'|s} \pi(s'|s) V^A(s').$$

With long-term bonds and the risk of default, it is not obvious that the government may not itself prefer autarky. Hatchondo et al. (2012) show, for example, that with long-duration bonds, the introduction of fiscal or debt limits can be beneficial from the perspective of the government. Such constraints limit the negative incentive effects that arise from long-term financing.[18] Nevertheless, one can show that even though a government may benefit from self-imposed debt limits, a debt limit of 0 is too much, and would never be optimal from its perspective:

Lemma 2 *In any Markov equilibrium, and for any maturity* δ, $V(0, s) \geq V^A(s)$ *for all* $s \in S$.

Proof In the Appendices. □

The lemma establishes that *a government will never choose to shut itself out of sovereign debt markets completely*.

[17] An alternative reference would be to allow the government to save but not borrow. Doing this in principle will raise the value of financial constraints from the perspective of the households, biasing our current results against finding welfare gains from financial market access.

[18] See Aguiar et al. (2019) for a discussion of maturity choice under lack of commitment.

4 The Welfare Gains and Losses of Financial Market Access

We now assess the potential welfare gains and losses due to financial market access. We do this under a sequence of calibrations used in the quantitative literature.

We define the autarky welfare (starting from zero debt) for the representative household as the solution to the following functional equation:

$$W^A(s) = u(y(s)) + \beta_{\mathrm{H}} \sum_{s'|s} \pi(s'|s) W^A(s').$$

As before, the disagreement between the government and the households arises when $\beta_{\mathrm{G}} \neq \beta_{\mathrm{H}} = e^{-\rho_{\mathrm{H}}}$.

Given an equilibrium policy function $\mathcal{B}$ and associated equilibrium prices, we can compute the welfare of the representative household:

$$\begin{aligned} W(b,s) = {} & u(y(s) - b + q(\mathcal{B}(b,s),s)(\mathcal{B}(b,s) - \delta b)) \\ & + \beta_{\mathrm{H}} \sum_{s'|s} \pi(s'|s) \mathbb{1}_{(V(\mathcal{B}(b,s),s') \geq \underline{V}(s'))} W(\mathcal{B}(b,s),s') \\ & + \beta_{\mathrm{H}} \sum_{s'|s} \pi(s'|s) \mathbb{1}_{(V(\mathcal{B}(b,s),s') < \underline{V}(s'))} \underline{W}(s') \end{aligned}$$

$$\underline{W}(s) = u(y^D(s)) + \beta_{\mathrm{H}} \sum_{s'|s} \pi(s'|s) \big(\theta W(0,s') + (1-\theta)\underline{W}(s')\big),$$

where $V, \underline{V}, q, \mathcal{B}$ represents the components of a Markov equilibrium of the economy when the government has access to sovereign debt markets.

Just as we did before, we can then compute the welfare gains, λ, of having financial market access. With CRRA utility, we have:

$$1 + \lambda = \begin{cases} e^{\{(1-\beta_{\mathrm{H}})(\mathbb{E}_s[W(0,s)] - \mathbb{E}_s[W^A(s)])\}}; & \text{if } \sigma = 1 \\ \left[\frac{\mathbb{E}_s[W(0,s)]}{\mathbb{E}_s[W^A(s)]} \right]^{\frac{1}{1-\sigma}}; & \text{if } \sigma \neq 1, \end{cases}$$

where the expectation operator is taken over the ergodic distribution of the Markov process s.[19]

4.1 A Welfare Decomposition

In this section, we describe the decomposition that we use to analyse the welfare losses across different calibrations.

For a given equilibrium, we let h_t denote a history of exogenous states and exclusion up to (and including) time t starting from *an initial debt equal to zero*. In

[19] In principle, we can take the expectation over any initial distribution. We choose the ergodic for simplicity of exposition.

particular, a history is given by $h_t = \{(s_0, d_0), (s_1, d_1), \ldots, (s_t, d_t)\}$, where s_t represents the exogenous state in period t and d_t is an indicator variable that takes value of 1 if the country is excluded from financial markets in period t, and 0, otherwise. In an abuse of notation, we let $\pi(h_t|h_0)$ denote the probability that history h_t is realized starting from state h_0. Because default, and as result exclusion, is endogenous, $\pi(h_t|h_0)$ is equilibrium dependent. A first result is that in any Markov equilibrium, the consumption level at time t can be written as function of h_t:

Lemma 3 *Consider a Markov equilibrium, $V, \underline{V}, q, \mathcal{B}$. Starting from a state s_0, the consumption realization at time t is such that $c_t = C(h_t)$ for a function C where h_t is the history of exogenous states and exclusion and $h_0 = (s_0, d_0)$ with $d_0 = 0$.*

Proof Suppose that at time t, the country is excluded. In this case, consumption equals y^D, which is just a function of the current exogenous state, s^t. And thus, for those histories, the result of the lemma follows.

Suppose that at time t, the country is not excluded. Let $\hat{t}(t)$ denote the time before t where the country was last excluded. We let $\hat{t}(t) = -1$ if the country has never been excluded before. From period $\hat{t}(t)$ to t, the country has not defaulted, and it starts the equilibrium with zero debt in period $\hat{t}(t) + 1$. Using that the equilibrium debt policy $\mathcal{B}$ is a function of the previous level of debt and the exogenous state, we can iterate from $\hat{t}(t) + 1$ starting with zero debt, and obtain that the level of consumption at t is just a function of the evolution of the exogenous state since $\hat{t}(t) + 1$. □

Note that in the above proposition, the function C is equilibrium dependent. Given this result, we can compute the welfare in any equilibrium by using the associated function C and the evolution of the history:

$$W(0) = \sum_{s_0} \pi^\infty(s_0) \sum_{t=0}^{\infty} \sum_{h_t} \pi(h_t|h_0 = (s_0, 0))\beta_{\rm H}^t u(C(h_t)),$$

with $h_0 = (s_0, 0)$, where $\pi^\infty(s_0)$ equals the ergodic distribution over the initial exogenous state s_0.[20]

Using the consumption function C, we can compute the following consumption paths:

$$\begin{aligned} c^{ND}(h_t) &= (1 - d_t)C(h_t) + d_t y(s_t), \\ \bar{c}^{ND}(t) &= \sum_{h_0} \pi^\infty(s_0) \sum_{h_t} \pi(h_t|h_0 = (s_0, 0))c^{ND}(h_t), \end{aligned}$$

where (s_t, d_t) represents the last realization in the history h_t.

We complement the above definitions with

[20] Note that we start the history from no exclusion in the initial period. The reason is that a government never defaults with zero debt.

$$\bar{c}_A(t) = \sum_{s_0} \pi^\infty(s_0) \sum_{h_t} \pi(h_t | h_0 = (s_0, 0)) y(s_t)$$
$$= \sum_{s_0} \pi^\infty(s_0) \sum_{s^t} \pi(s^t | s_0) y(s_t) = y^\infty,$$

where y^∞ is the ergodic mean of the output process. We can define the following objects:

$$W^{ND}(0) = \sum_{s_0} \pi^\infty(s_0) \sum_{t=0}^{\infty} \sum_{h_t} \pi(h_t | h_0 = (s_0, 0)) \beta_{\mathrm{H}}^t u(c^{ND}(h_t))$$
$$\overline{W}^{ND}(0) = \sum_{t=0}^{\infty} \beta_{\mathrm{H}}^t u(\bar{c}^{ND}(t)).$$

Let us briefly discuss what each of these value functions mean. $W^{ND}(0)$ computes the welfare associated with an equilibrium where *the output loss from default has been eliminated*. In states where the country is excluded, the consumption process has been adjusted to equal the associated output $y(s_t)$ without the default costs. $\overline{W}^{ND}(0)$ calculates the welfare associated with the *mean consumption path* without default. This value function thus eliminates the variability of the consumption path (exclusive of the default costs).

We can then decomposed the welfare gains λ into three different factors: λ_{D}, λ_{V} and λ_{T} (here done for $\sigma \neq 1$):

$$(1+\lambda) = \left[\frac{W_0}{W^A}\right]^{\frac{1}{1-\sigma}}$$
$$= \underbrace{\left[\frac{W_0}{W^{ND}(0)}\right]^{\frac{1}{1-\sigma}}}_{1+\lambda_{\mathrm{D}}} \times \underbrace{\left[\frac{W^{ND}(0)}{\overline{W}^{ND}(0)} \times \frac{\overline{W}^A}{W^A}\right]^{\frac{1}{1-\sigma}}}_{1+\lambda_{\mathrm{V}}} \times \underbrace{\left[\frac{\overline{W}^{ND}}{\overline{W}^A}\right]^{\frac{1}{1-\sigma}}}_{1+\lambda_{\mathrm{T}}}$$
$$= (1+\lambda_{\mathrm{D}}) \times (1+\lambda_{\mathrm{V}}) \times (1+\lambda_{\mathrm{T}}).$$

The first factor λ_{D} captures the role of *default costs* and computes the percentage increase in consumption necessary to compensate the households for the reduction in output due to default. Once we have removed the default costs, λ_{V} captures the additional role of the *variability* of consumption in the welfare comparison between the allocation with market access and autarky. Finally, given that bond prices are actuarially fair,

$$\sum_{t=0}^{\infty} R^{-t}(\bar{c}_t^{ND} - y^\infty) = 0.$$

As a result, the last term λ_{T} captures the welfare effects generated by the tilting of consumption away from the average constant endowment.

In terms of comparison with our simple model, λ_{T} and $\hat{\lambda}$ are both capturing the welfare role of the tilting of expenditures. The values of λ_{D} and λ_{V} are the additional terms that uncertainty generates. They arise because of the potential emergence of default costs in equilibrium and variability of expenditures around the mean endowment.

We now use this decomposition to understand the welfare losses from market access in several models.

4.2 Welfare Losses Across Different Models

In what follows, we will narrow attention to three different calibrations of the Eaton and Gersovitz (1981) model. All the simulations are done with a period equal to a quarter. To keep the results of each of these calibrations consistent, we set certain parameters the same across them. In particular, in all of the simulations below, the utility parameter σ is set to a value of 2 (which corresponds to the value used in each of the papers we focus on). The income process in all of the exercises is set to one that approximates the quarterly income process for Argentina.[21] However, we adjust the maturity parameter δ; re-entry parameter θ; the default cost process $\{y^D(s)\}$; the discount factor of the government β_{G}; and the risk-free real interest rate R, across the alternative simulations. The parameter values used are described in Table 1. Some key moments from the simulations are shown in Table 2.

The first two calibrations consider an environment with one-period bonds, so $\delta = 1$ in each of them. Critically, the default cost process and the government discount factor are different in both of them.

4.3 Losses in Aguiar and Gopinath (2006)'s Calibration

We start with the transitory-shock version of Aguiar and Gopinath (2006), henceforth AG, which is the closest to the original EG framework. The primary difference is that AG introduce a linear cost of default, which is necessary to support non-trivial amounts of debt in equilibrium.

Following AG, we set the international risk-free (quarterly) gross interest rate to $R = 1.01$ and the (quarterly) re-entry parameter to $\theta = 0.10$. The (quarterly) discount factor of the government equals $\beta_{\mathrm{G}} = 0.8$, which generates an annual discount rate of $\rho_{\mathrm{G}} = 0.89$. The endowment process under default is reduced in each state by 2%, that is, $y^D(s) = 0.98y(s)$.

[21] Our specification of the income process is the same as the specification in Chatterjee and Eyigungor (2012), including both a persistent component and a transitory component. In all the simulations we perform, we similarly set the transitory endowment component to its lowest value in a period where a default occurs. The transitory component is small and needed to facilitate the convergence of the numerical computations in the long-term bond case. Its presence is not necessary for the one-period bond calculations (and its effects there are not significant). We refer the reader to Chatterjee and Eyigungor (2012) for more details.

With these parameter values, we solve for the Markov equilibrium numerically. The model's debt-to-(annual) output ratio converges (conditional on no default) to an average level of 0.06, a number very close to the 0.0625 obtained in Aguiar and Gopinath (2006).[22]

Given the simulation results, we compute the counterpart to Fig. 2, which is presented in Fig. 3. The figure shows the consumption equivalent gains of a household with an annual discount rate ρ_{H} of giving market access to the government. This is shown in the solid dark line. The solid light line in the figure is the prediction from the deterministic model using the formula (3), where we used annualized parameters corresponding to the ones in the calibration. Accordingly, we set $r = 4\log(R) = 0.04$, $\rho_{\mathrm{G}} = -4\log(\beta_{\mathrm{G}}) = 0.89$, and $\overline{b}/y = 0.06$.

A surprising finding is how well our previous back-of-the-envelope exercise is able to very accurately capture the welfare losses from market access in this calibration. Both lines in Fig. 3 are right on top of each other.

This calibration confirms the general message of the back-of-the-envelope exercise: For a large range of their discount factors, the households are better off under a regime where the government has access to international financial markets, even when the inter-temporal disagreement between the household and its government is large. As shown in Fig. 3, it is only when the household discount rate approaches the international discount rate that financial autarky becomes an attractive choice. And even for values close the market discount rate (but higher), the losses generated by financial market access are quite small—achieving its highest value of 0.053% of consumption for $\rho_{\mathrm{H}} = r$.

Figure 4 shows the welfare decomposition and displays the λ_{T}, λ_{V}, and λ_{D} components. The figures shows that the variation in the welfare gains is fully accounted by the λ_{T} term, and the λ_{V} and λ_{D} components are practically zero. This is the result of this calibration missing on one particular key dimension: the default probability is small (on the order of a 0.3 % annual rate of transiting from a good credit standing to the default state). As stressed in the original AG paper, defaults in this calibration are rare events. As we explore in the next subsections, the presence of uncertainty coupled with a significant risk of default *in equilibrium* can significantly alter the balance between market access and financial autarky, tilting towards the second.

4.4 Losses in Arellano (2008)'s Calibration

The Arellano (2008) builds on the same EG platform as AG, but has a richer model of default. Arellano's major departure from EG and AG is the introduction of a *non-linear* state-dependent default cost. In particular, let

$$y^D(s) = \min\{y(s), \hat{y}\}.$$

[22] We compare our results to Aguiar and Gopinath (2006)'s transitory shocks model (Model I). They obtained a 0.25 (quarterly) debt-to-output ratio. The only difference between our numerical exercise and their transitory shocks model is the specification of output process.

Table 1 Parameter specifications used (quarterly values)

Parameter	AG	Arellano	CE
δ	1.000	1.000	0.9500
β_{G}	0.800	0.953	0.9540
θ	0.100	0.282	0.0385
r	0.010	0.017	0.0100
$\tilde{y}^D(s)$	$0.98\tilde{y}(s)$	$\min\{0.969\mathbb{E}\tilde{y}(s), \tilde{y}s)\}$	$\tilde{y}(s) - \max\{-0.188\tilde{y}(s) + 0.246\tilde{y}(s)^2, 0\}$

The values of $\tilde{y}(s)$ and $\tilde{y}^D(s)$ refer to the permanent component of the output process. See footnote 21 for an explanation. AG refers to Aguiar and Gopinath (2006). Arellano refers to Arellano (2008). CE refers to Chatterjee and Eyigungor (2012)

Table 2 Model statistics (annualized values)

Statistic	AG (%)	Arellano (%)	CE (%)
Ergodic default frequency	0.30	2.86	5.62
Ergodic default mass	0.74	2.49	27.02
Mean ergodic debt/GDP	6.02	0.97	19.78
Limiting mean debt/GDP on paths w/o default	6.02	1.10	22.11
Conditional mean default cost/persistent GDP	2.00	0.50	4.73
Unconditional mean default cost/persistent GDP	0.01	0.01	1.27

The debt-to-output ratio in CE is computed following the transformation described in footnote 25 and the definition in Eq. (4). The "limiting mean debt/GDP on paths without default" is the mean debt-to-output ratio that the economy converges to conditional on equilibrium paths where default does not occur, starting from zero debt and from the ergodic endowment distribution. The mean default costs are computed conditional on being in default (the row labelled "conditional mean default cost/persistent GDP") and unconditional on default status. The latter is the former multiplied by the fraction of periods spent in default (the row labelled "ergodic default mass"). AG refers to Aguiar and Gopinath (2006). Arellano refers to Arellano (2008). CE refers to Chatterjee and Eyigungor (2012)

The asymmetry in this default cost turns out to have significant quantitative implications. The asymmetry implies that for low-endowment realizations, that is, for $y(s) < \hat{y}$, defaulting generates no immediate additional drop in output. The only deadweight costs arise from the lack of access to borrowing and saving (which are quantitatively small, as demonstrated by a simple calculation in AG), and the possibility of a future output costs if the endowment transits to a higher level in the future prior to re-entry (which is also mitigated by the persistence in the endowment process).

For $y(s) > \hat{y}$, the output costs equal $y(s) - \hat{y}$. Hence, default in high-endowment states is punished much more harshly. As discussed in Arellano (2008), this specification of default costs allows the model to generate a high default probability in equilibrium.

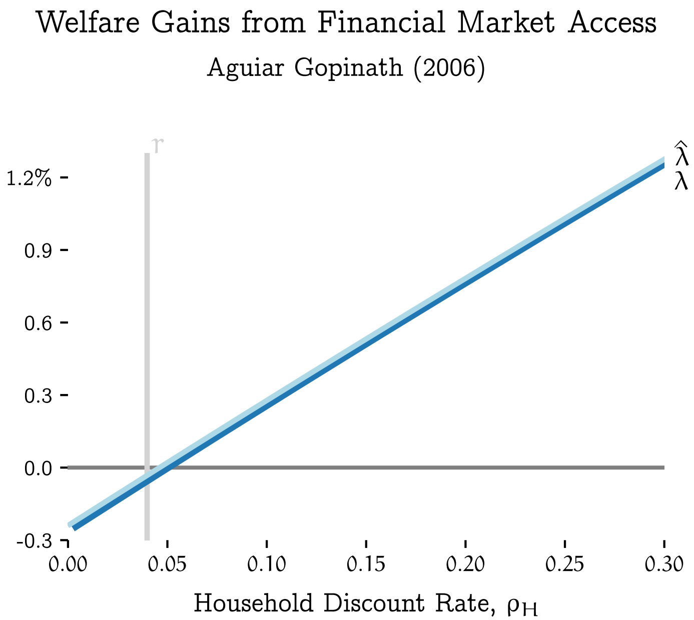

Fig. 3 The x axis is the annualized household discount rate. The y axis represents the welfare gains in percentage points of consumption. The solid dark line is the results from the Aguiar and Gopinath (2006) calibration. The solid light line is the result from using formula (3). The vertical line corresponds to the (annualized) international interest rate

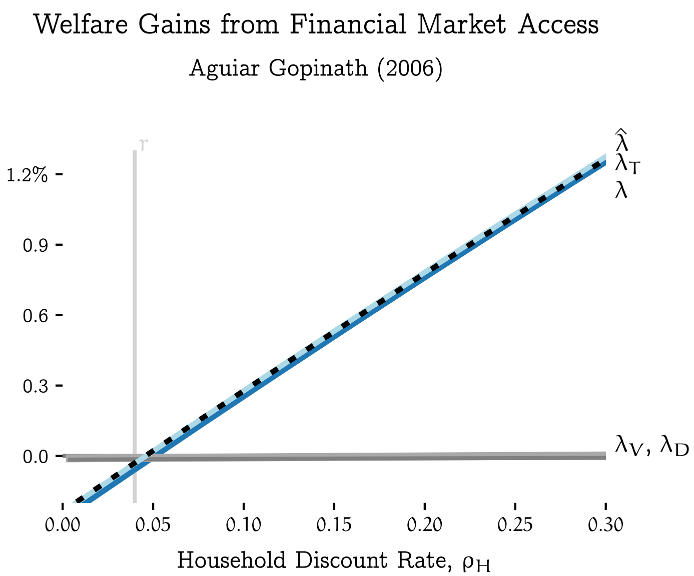

Fig. 4 The x axis is the annualized household discount rate. The y axis represents the welfare gains in percentage points of consumption. The vertical line corresponds to the (annualized) international interest rate

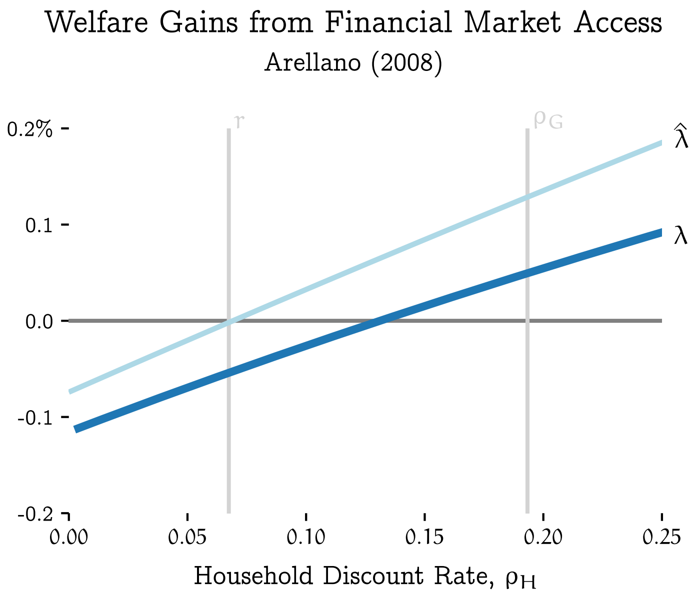

Fig. 5 The x axis is the annualized household discount rate. The y axis represents the welfare gains in percentage points of consumption. The two vertical lines correspond to the (annualized) international interest rate and the government discount rate

The introduction of this flexible default specification reduces the government discount rate needed to match the data. We set the (quarterly) discount factor to $\beta_G = 0.953$, the value obtained in Arellano (2008). Similarly to that paper, we set the risk-free (quarterly) interest rate to $R = 1.017$, the re-entry probability to $\theta = 0.282$ (quarterly) and let the critical income cut-off be given by $\hat{y} = 0.969\mathbb{E}y(s)$, where $\mathbb{E}y(s)$ represents the ergodic mean of the output process.

Figure 5 presents the computation of the welfare gains, λ, as a function of the household discount rate for this calibration. As before, the dark line is the calibration results, while the light line represents the gains obtained directly from equation (3). For the latter, we set $r = 4\log(R) = 0.067$, $\rho_G = -4\log(\beta_G) = 0.193$, and $\overline{b}/y = 0.011$, the equivalent annualized parameter values. Again, similarly to the results in the previous model, our back-of-the-envelope calculation does a good job at capturing the order of magnitude of losses generated by financial market access.

Interestingly, and different from the previous calibration, the balance has tilted towards financial autarky. As shown in Fig. 5, for annual discount rates below 0.13, the household would strictly prefer that the government has no access to international financial markets. Given that a 13% annual discount rate is above the discount rate usually assumed for households in economic models, Fig. 5 makes a stronger case for eliminating access to external borrowing than the previous calibration.

This calibration generates a more substantial default risk in equilibrium. In the ergodic distribution, the probability of switching from a good credit standing to a default state is around an annual rate of 2.8%, a much higher number than in the previous one. Part of the reason this occurs has to do with the flexible default cost function, which allows for default to occur at a much lower cost in certain endowment states. We now use our decomposition to analyse whether the presence of default risk is the key element explaining the disagreement with respect to market access.

Figure 6 plots the decomposition of the welfare gains for different levels of the household discount factor. As can be seen, λ_T is close to $\hat{\lambda}$ – our simple benchmark exercise captures quite well the welfare effects generated by the tilting of the consumption profile away from the autarkic endowment. Interestingly, the figure shows that the default costs are not important for the household's welfare computation, and the λ_D term remains close to zero. Default, in this environment, is occurring mostly in states where default is not too costly.

Most of the difference between the prediction of our back-of-the-envelope exercise and the calibration arises from the increase in the variability of consumption, the λ_V term. That is, it is the desire to eliminate the additional variability of consumption generated by market access that make households prefer shutting down debt market access. As emphasized in Arellano (2008), sovereign debt induces the government to follow a pro-cyclical spending pattern—increasing expending in high-endowment states (when default costs are high and interest rates low) while decreasing expending in low-endowment states (when default costs are low, and interest rates high and sensitive to debt levels).

Finally, although Fig. 5 strengthens the case against external borrowing, the welfare magnitudes involved remain small. The welfare losses from financial market access at $\rho_G = r$ equal 0.054% of consumption. (This is the highest possible number for discount rates weakly above the interest rate.) The difference in the variability of consumption is not large enough to generate large welfare effects.[23] In addition, this calibration does not generate significant amounts of sovereign lending, and as our simple exercise suggests, the implied welfare effects from tilting should be small.

In order to deal with the shortcomings of one-period bond models, the literature has incorporated long-duration bonds into the environment. The presence of long-term debt allows the model to match a higher external debt-to-output ratio (closer to those observed in emerging markets). They also introduce an additional inefficiency due to debt dilution. As we will show next, these forces significantly strengthen the case against external government borrowing.

4.5 Long-Duration Bonds

Hatchondo and Martinez (2009) and Chatterjee and Eyigungor (2012), henceforth CE, were the first to extend the quantitative EG model to an environment with long-term bonds. We follow the parameter values used in Chatterjee and Eyigungor (2012). The (quarterly) value of the maturity parameter is set to $\delta = 0.95$, which generates an average bond maturity of 5 years. The (quarterly) real interest rate is set to $R = 1.01$. The re-entry parameter is set to a (quarterly) value of $\theta = 0.0385$

[23] This last is related to the cost of business cycles. See Lucas (1987).

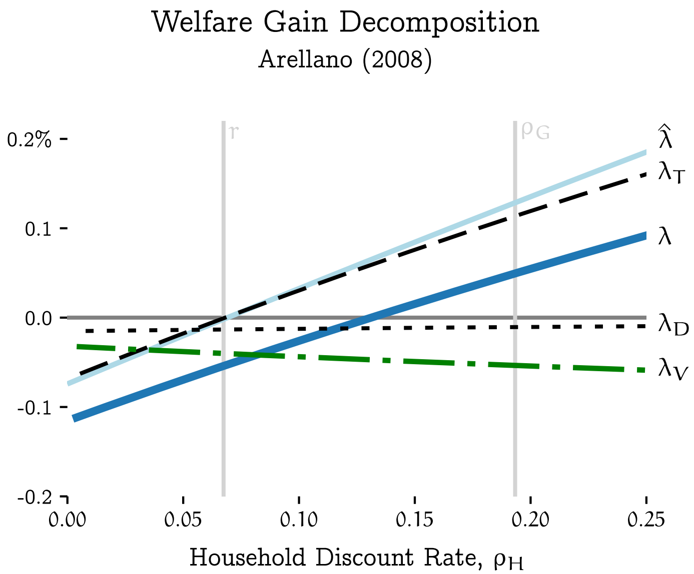

Fig. 6 The x axis is the annualized household discount rate. The y axis represents the welfare gains in percentage points of consumption. The two vertical lines correspond to the (annualized) international interest rate and the government discount rate

(which generates a longer exclusion period than the previous two exercises). Following Arellano (2008), CE use a nonlinear cost of default; specifically, the output process during exclusion is given by the following specification:

$$y^D(s) = y(s) - \max\{0, d_0\tilde{y}(s) + d_1\tilde{y}(s)^2\},$$

where $\tilde{y}(s)$ refers to the persistent component of the output process, and where $d_0 = -0.18819$ and $d_1 = 0.24558$.[24]

The (quarterly) discount factor of the government is $\beta = 0.954$, implying an annual government discount rate of $\rho_G = 0.188$. To calculate the debt-to-output ratio to be used in our back-of-the-envelope exercise, consider the following. Suppose that the bond was risk-free; in this case, its price would be $q^\star \equiv \frac{1}{1+r-\delta}$. Now, let us calculate the budgetary cost of rolling over the debt forever. That is, setting $b' = b$, from the budget constraint we get that

[24] In Chatterjee and Eyigungor (2012), the output process is the sum of two components: $y(s) = \tilde{y}(s) + m(s)$, where $\tilde{y}(s)$ follows a persistent Markov chain and $m(s)$ is an i.i.d. process. In the period where default is triggered, the transitory component is set to its lowest level in the support, while it reverts back to its normal stochastic process in subsequent periods. See the related discussion in footnote 21.

$$c - y(s) = -b + q^\star(b' - \delta b) = -b + q^\star(1-\delta)b$$
$$= r\frac{b}{1+r-\delta}.$$

Thus, servicing an amount b of long-duration bonds is equivalent to servicing an amount $b/(1+r-\delta)$ of one-period bonds (under risk-free pricing). We thus use

$$\frac{\text{debt}}{y} \equiv \frac{b/y}{1+r-\delta} \tag{4}$$

as our equivalent debt-to-output ratio in our back-of-the-envelope exercise. Conditional on no default, this value converges in the simulation to an annual debt-to-output ratio of 0.221.[25]

As shown in Table 2, this calibration of the model delivers a higher debt-to-output ratio and also a higher probability of default. In this calibration, the annual probability of default is 5.7%.

Figure 7 shows the corresponding values of λ for this calibration. First, the welfare gains from denying access are now an order of magnitude larger. Part of the reason is that the debt-to-output ratio that this model generates is also larger than the previous ones. And second, disagreement about financial market access arises for a large range of discount rates for the households. Different from the prediction of the back-of-the-envelope exercise, households with an annual discount rate less than 0.10 would strictly prefer that their government have no access to financial markets. The welfare losses now reach 1.1% of consumption when the household discount rate equals the market interest rate.

To understand why the welfare magnitudes and the range of disagreement are larger in this calibration, we perform the same decomposition as before. As a first step, we compared the variability term λ_V in this simulation with the one in the Arellano (2008)'s calibration. This comparison is shown in Fig. 8. As can be seen, these terms are similar to each other in magnitude. But there is an important difference. The variability term was the main driver of the disagreement with respect to market access in the Arellano (2008) calibration. Even though this term is slight larger (in absolute value) in the CE calibration, it is almost insignificant for the welfare analysis in this case. The reason is that both the welfare effects

[25] Chatterjee and Eyigungor (2012) use a different specification for the long-term bond. They assume that a given bond matures with an idiosyncratic probability λ, in which case it pays a unit to its holder. If the bond does not mature, it pays a coupon z. The budget constraint (using hats to represent these alternative bonds) is now

$$c = y(s) - (\lambda + (1-\lambda)z)\hat{b} - \hat{q}(s, b')(\hat{b}' - (1-\lambda)\hat{b})$$

and the pricing equation

$$\hat{q} = \mathbb{E}\left[1(\text{No default})\frac{\lambda + (1-\lambda)(z + \hat{q}')}{R}\right]$$

This is equivalent to our formulation with the following change in variables: $q = \hat{q}/(\lambda + (1-\lambda)z)$, $b = (\lambda + (1-\lambda)z)\hat{b}$, $\delta = 1 - \lambda$.

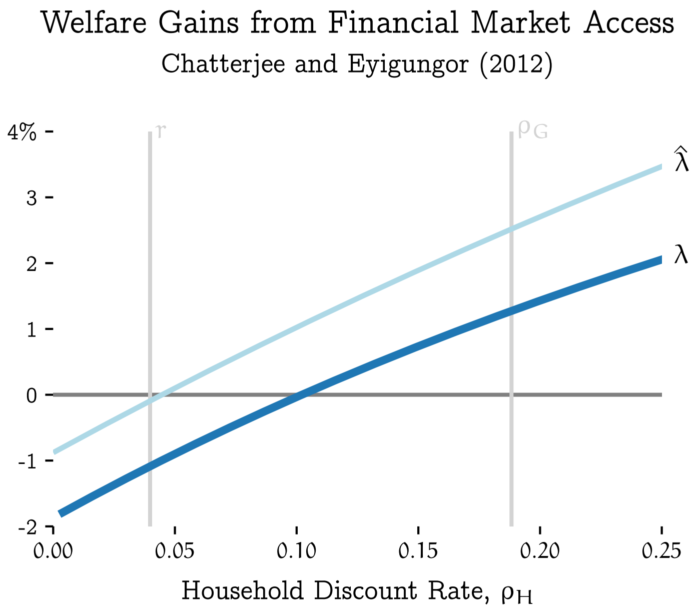

Fig. 7 The *x* axis is the annualized household discount rate. The *y* axis represents the welfare gains in percentage points of consumption. The two vertical lines correspond to the (annualized) international interest rate and the government discount rate

from the tilting of expenditures (λ_T) and the welfare effects generated by the equilibrium default costs (λ_D) are of a larger magnitude. This full decomposition for the CE model is shown in Fig. 9.

As mentioned above, the variability term is not important. (λ_V is close to zero as compared to the other terms.) The tilting of the consumption allocation away from the its autarkic value generates no disagreement with respect to market access. This is the same as in our baseline exercise and in the other two calibrations. That is, without default costs (just focusing in $\lambda_T + \lambda_V$), slightly impatient households strictly prefer their impatient government to have access to international financial markets. Even when they would prefer the opposite, the losses are an order of magnitude smaller than those found when the default costs are taken into account (λ_D). Thus, the disagreement between the households and the government with regard to market access is accounted for in this model by default costs – the households would like to stop the government from borrowing so as to eliminate future default events.

The intuition is as follows. In sovereign debt models, the government borrows from international creditors because it is impatient. This behaviour generates a front-loading of expenditures and a potential increase in variability of consumption. Both of these effects are quantitatively small. However, by borrowing, the government also exposes the country to future default events. These default episodes are costly and significant enough in terms of welfare for the households to prefer no market access for a wide range of discount factors.

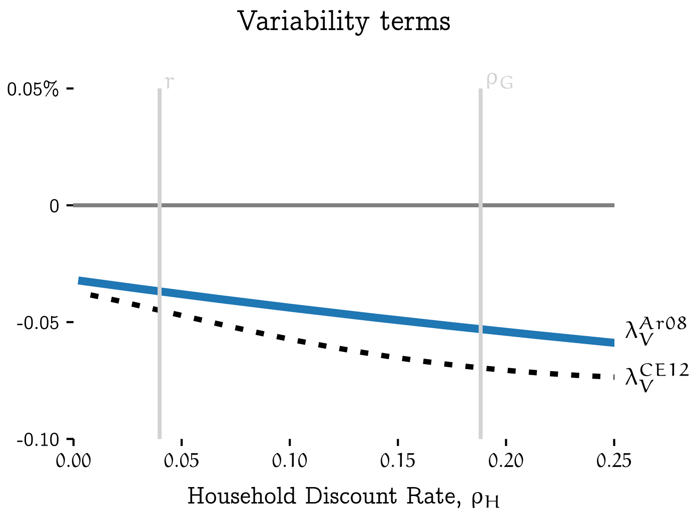

Fig. 8 The *x* axis is the annualized household discount rate. The *y* axis represents the welfare gains in percentage points of consumption. The two vertical lines correspond to the (annualized) international interest rate and the government discount rate

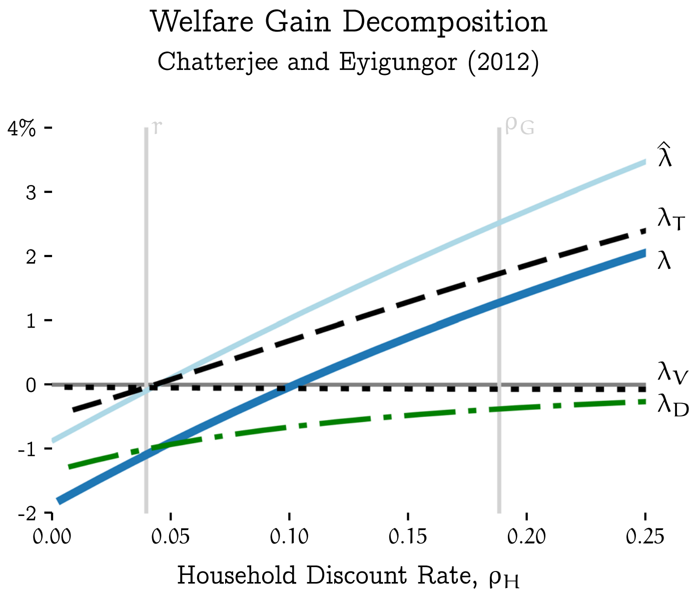

Fig. 9 The *x* axis is the annualized household discount rate. The *y* axis represents the welfare gains in percentage points of consumption. The two vertical lines correspond to the (annualized) international interest rate and the government discount rate

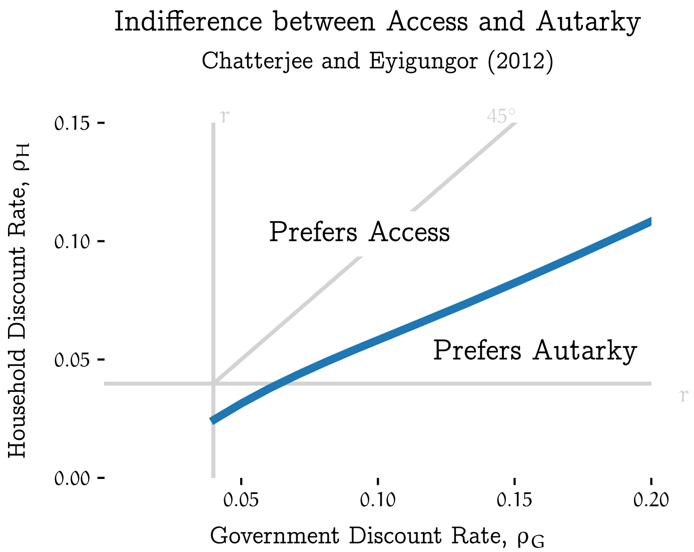

Fig. 10 The solid line represents the (annualized) household discount rate (y axis) that keeps a household indifferent between market access or not, for different (annualized) government discount rates (x axis). The rest of the parameters are as in the CE calibration

We now replicate Fig. 1 in the context of the CE calibration. That is, keeping all other parameters constant, we compute, for different values of the government discount rate ρ_{G}, the household discount rate, ρ_{H}, that will make the household indifferent between market access or not. Recall from Fig. 1 that the back-of-the-envelope calibration suggested that such household discount rate would barely change and would remain close to the market interest rate. For the CE calibration, the results are quite different.

Figure 10 replicates the indifference calculation in Fig. 1 performed this time with the CE calibration. As can be seen, the indifference line is much steeper than before. There is a wider range of household discounts factors that disagree with the government access to external sovereign debt markets. Even more, such disagreement quantitatively increases as the government becomes more impatient (that is, as its discount rate increases).[26]

In Appendix 3, we further decompose how the government and households value the default costs differently at different discount rates.

[26] It is important to note, however, that this indifference line does not fully correspond to that of Fig. 1. In that figure, the debt-to-output ratio was kept constant. In Fig. 10, as we change the government discount factor, the debt-to-output ratio in equilibrium changes.

5 Conclusion

In this paper, we have gathered some lessons from the quantitative sovereign debt literature regarding the costs of external sovereign borrowing. We started with a deterministic debt model in which the exogenous borrowing limit pins down the losses to the households if they disagree with the government about time preference.

The back-of-the-envelope exercise turns out to be a very good predictor of the welfare gains from having access to financial markets in the richer, quantitative sovereign debt models used in the literature, when calibrated such that the default probability is small (as in Aguiar and Gopinath 2006). We showed that the inter-temporal distortion in spending is not quantitatively large enough to generate a significant level of disagreement between the households and the government regarding access to external borrowing in this case. And, in addition, the welfare magnitudes involved are not large, a fact generated by the low amount of external debt involved.

The introduction of a significant level of default risk matters for this result. When default risk is high, but the default costs generated in equilibrium are low (as in Arellano 2008), the additional variability of consumption that access to external markets generates is sufficient to drive a disagreement between households and its government with regard to market access.[27] However, the welfare magnitudes involved remain small, as the amount of debt is not large and default; although it occurs in equilibrium, it generates small deadweight losses.

However, using the latest cohort of sovereign debt models (for example, Chatterjee and Eyigungor 2012), which include long-term bonds as well as a more flexible specification of the output costs, we find that the level of disagreement is large. Relatively impatient households living in this model will prefer a situation where their government cannot access external sovereign markets. The magnitudes of the welfare gains that such a market shutdown would generate are significant. We showed that this result is driven by the exposure to costly future defaults that the government generates from its excess borrowing.

Our conclusion is that the maturity of the debt, as well as the shape of the default costs, are critical to evaluate the benefits of sovereign debt market access. We can observe very well the first one. But there is greater uncertainty regarding the second, as the costs of default are usually inferred from other calibration targets. But the existence of default costs, which occur in equilibrium, can significantly strengthen the case against access to external sovereign debt markets.

Along the way, we developed a decomposition of the welfare costs of market access in the context of quantitative sovereign debt models. This decomposition separates the welfare cost into three terms: the front-loading of expenditures, the excess variability of expenditures, and the deadweight losses from default. Our decomposition also shows that the recent quantitative sovereign debt models are not necessarily models of risk sharing (even though the original work of Eaton and Gersovitz (1981) is), as the

[27] This excess variability of consumption with respect to income is one of the quantitative successes of the sovereign debt model. The other is the counter-cyclicality of the trade balance, which is related: the country borrows more in good times generating a more volatile consumption process.

contribution of the additional variability of expenditures to welfare is negative in all the exercises we conducted.

Just as we did in the introduction, we highlight that the models we have used in this paper are missing certain dimensions that may alter our results. First, our analysis is missing a domestic investment decision, as there is no domestic capital. On this, it should be possible to use the recent results of Gordon and Guerron-Quintana (2018) that incorporate investment and domestic capital to recompute our calculations. Second, we focus attention on the disagreement about just one parameter (impatience), and did not consider other channels (such as disagreement about the composition of expenditure, risk aversion, or the elasticity of the inter-temporal substitution). Finally, the Eaton and Gersovitz (1981) framework we consider ignores the possibility of self-fulfilling runs. We leave the question of how these different mechanisms affect out results for future research.

Appendix 1: Proof of Lemma 1

The functional form describe in Eq. (3) arises from the solutions to the welfare functions obtained in the text. To see that this expression is strictly increasing in ρ_{G}, consider the case where $\rho \neq 1$.

First note that $\overline{b} > 0$ and $\rho_{\mathrm{G}} > r$ imply that $T > 0$.

Let $\kappa \equiv (\rho_{\mathrm{G}} - r)\frac{1-\sigma}{\sigma} \neq 0$. Note that $\rho_{\mathrm{G}} > r$ implies that κ inherits the sign of $1 - \sigma$, given $\sigma > 0$. Whether λ increases with ρ_{H} then depends on whether

$$\frac{\rho_{\mathrm{H}} e^{\kappa T} + e^{-T\rho_{\mathrm{H}}}\kappa}{\kappa + \rho_{\mathrm{H}}}$$

strictly increases when $0 < \sigma < 1$ ($\kappa > 0$) and decreases when $\sigma > 1$ ($\kappa < 0$).

The derivative with respect to ρ_{H} is

$$-\kappa \frac{e^{-T\rho_{\mathrm{H}}}(1 - e^{T(\kappa+\rho_{\mathrm{H}})} + T(\kappa + \rho_{\mathrm{H}}))}{(\kappa + \rho_{\mathrm{H}})^2}.$$

Note that this derivative is continuous for all $\rho_{\mathrm{H}} > 0$, equating $\kappa\frac{1}{2}e^{T\kappa}T^2$ at $\rho_{\mathrm{H}} = -\kappa$. Hence, the derivative at $\rho_{\mathrm{H}} = -\kappa$ is nonzero (as $T > 0$) and inherits the sign of κ.

It suffices to check that

$$1 - e^{T(\kappa+\rho_{\mathrm{H}})} + T(\kappa + \rho_{\mathrm{H}}) < 0$$

for the rest of the domain, $\rho_{\mathrm{H}} \neq -\kappa$. But this follows as the above is a strictly concave function of ρ_{H} (given $T > 0$), with a maximum of 0 at $\rho_{\mathrm{H}} = -\kappa$.

The proof for the case of $\sigma = 1$ is simpler and left to the reader.

Appendix 2: Proof of Lemma 2

From the value function of government, starting from any s where $b = 0$, it follows that it is feasible to set $b' = 0$ (independently of the price). In that case, such a strategy provides a lower bound to the value, and thus:

$$V(0,s) \geq u(y(s)) + \beta_{\mathrm{G}} \sum_{s'|s} \pi(s'|s) \max\{V(0,s'), \underline{V}(s')\}$$
$$\geq u(y^D(s)) + \beta_{\mathrm{G}} \sum_{s'|s} \pi(s'|s)(\theta\{V(0,s') + (1-\theta)\underline{V}(s')) = \underline{V}(s),$$

where the second inequality follows from $y^D(s) \leq y(s)$. Thus, $V(0,s) \geq \underline{V}(s)$, and

$$V(0,s) \geq u(y(s)) + \beta_{\mathrm{G}} \sum_{s'|s} \pi(s'|s) V(0,s').$$

Solving this recursion yields that

$$V(0,s) \geq V^A(s).$$

Appendix 3: Disagreement About the Default Value

The fact that λ_{D} is large in the CE model raises the question of to what extent does the government value default differently from households. We can compute λ_{D}^G in a manner similar to that of λ_{D}. Specifically, let V_0 denote the government's expected value conditional on zero debt. Let $V^{ND}(0)$ be constructed in the same was as $W^{ND}(0)$ but replacing β_{H} with β_{G}. We find $\lambda_{\mathrm{D}}^G = -0.38\%$. This is the value in Figure 9 at which λ_{D} intersects the $\rho_{\mathrm{H}} = \rho_{\mathrm{G}}$ vertical line. For $\rho_{\mathrm{H}} = r$, for example, λ_{D} is approximately -1%, or nearly three times λ_{D}^G.

This difference between λ_{D} and λ_{D}^G can be further decomposed. At the time of a default, the government and households disagree about the expected present value of post-default consumption due to the potential differences in discount factor. Suppose, instead, that households and the government valued the default state the same. That is, in the period of default, suppose households preferences become identical to the government's for all future periods (including post-re-entry). Using this alternative preference specification, we can compute $\hat{W}_0$ and $\hat{W}^{ND}(0)$. Specifically, let $T(h_t)$ denote the first time of default in history h_t, where we set $T(h_t) = t$ if no default has occurred as of t. Then,

$$\hat{W}_0 = \sum_{s_0} \pi^{\infty}(s_0) \sum_{t=0}^{\infty} \sum_{h_t} \pi(h_t|h_0 = (s_0, 0)) \beta_{\mathrm{H}}^{T(h_t)} \times \beta_{\mathrm{G}}^{t-T(h_t)} u\big(C(h_t)\big),$$

with $\hat{W}^{ND}(0)$ defined accordingly by replacing $C(h_t)$ with $c^{ND}(h_t)$ in the above. We can then define

$$1 + \hat{\lambda}_D \equiv \left(\frac{\hat{W}_0}{\hat{W}^{ND}(0)} \right)^{\frac{1}{1-\sigma}}.$$

This is the loss due to default costs, but evaluated such that the government and household preferences agree post-default. However, prior to default, the household discounts with a different discount factor. This leads to the following decomposition:

$$\frac{1 + \lambda_D}{1 + \lambda_D^G} = \frac{1 + \lambda_D}{1 + \hat{\lambda}_D} \times \frac{1 + \hat{\lambda}_D}{1 + \lambda_D^G}.$$

In Fig. 11, we plot the ratio on the left-hand side as well as the two components from the right-hand side as we vary ρ_H. In the figure, we see that the majority of the disagreement when the household is relatively patient is due to the first ratio on the right. This ratio captures the households disagreement post-default.

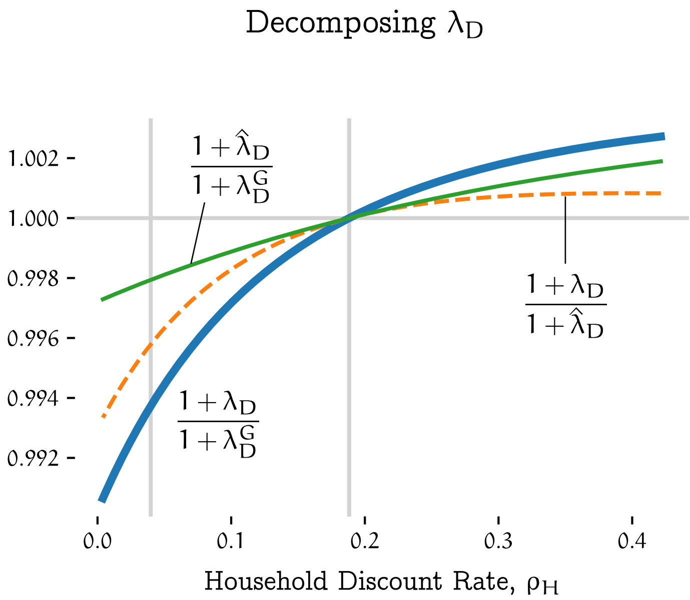

Fig. 11 The two vertical lines represent the market interest rate and the discount factor of the government in the CE calibration

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